1. Let
   
   \[ A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 & 2 \\ 3 & 1 & -2 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 1 & -1 \end{pmatrix}, \quad D = \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} \]

   a) Mark all the products that are defined, and give the dimensions of the result: \( AB, BA, ABC, ABD, BC, BC^T, B^T C, DC, D^T C^T \).

   b) Compute \( AB, A(3B + C), B^T A, A(BD), (AB)D \).

2. Let \( T_\gamma \) be the matrix of rotation by \( \gamma \) in \( \mathbb{R}^2 \). Check by matrix multiplication that \( T_\gamma T_{-\gamma} = T_{-\gamma} T_\gamma = I \).

3. Find the matrix of the orthogonal projection in \( \mathbb{R}^2 \) onto the line \( x_1 = -2x_2 \). **Hint:** What is the matrix of the projection onto the coordinate axis \( x_1 \)?

   You can leave the answer in the form of the matrix product, you do not need to perform the multiplication.

4. Find linear transformations \( A, B : \mathbb{R}^2 \to \mathbb{R}^2 \) such that \( AB = 0 \) but \( BA \neq 0 \).

5. Multiply two rotation matrices \( T_\alpha \) and \( T_\beta \) (it is a rare case when the multiplication is commutative, i.e. \( T_\alpha T_\beta = T_\beta T_\alpha \), so the order is not essential). Deduce formulas for \( \sin(\alpha + \beta) \) and \( \cos(\alpha + \beta) \) from here.