Homework assignment, March 12, 2004.

Due Monday, 3/15 (collected)

1. Find characteristic polynomials, eigenvalues and eigenvectors of the following matrices:

\[
\begin{pmatrix}
4 & -5 \\
2 & -3
\end{pmatrix}, \quad
\begin{pmatrix}
2 & 1 \\
-1 & 4
\end{pmatrix}, \quad
\begin{pmatrix}
1 & 3 & 3 \\
-3 & -5 & -3 \\
3 & 3 & 1
\end{pmatrix}.
\]

When it is possible, diagonalize the matrices, i.e. write them in a form \(A = SDS^{-1}\), where \(D\) is a diagonal matrix. Do not compute \(S^{-1}\).

2. Compute eigenvalues and eigenvectors of the rotation matrix

\[
\begin{pmatrix}
\cos \alpha & -\sin \alpha \\
\sin \alpha & \cos \alpha
\end{pmatrix}.
\]

Note, that the eigenvalues (and eigenvectors) do not need to be real.

3. Let

\[
A = \begin{pmatrix}
4 & 3 \\
1 & 2
\end{pmatrix}.
\]

Find \(A^{2004}\) by diagonalizing \(A\).

4. True or false:

a) Every linear operator in an \(n\)-dimensional vector space has \(n\) distinct eigenvalues.

b) If a matrix has one eigenvector, it has infinitely many eigenvectors.

c) There exists a square real matrix with no real eigenvalues.

d) There exists a square matrix with no (complex) eigenvectors.

e) Similar matrices always have the same eigenvalues.

f) Similar matrices always have the same eigenvectors.

g) The sum of two eigenvectors of a matrix \(A\) is always an eigenvector

h) The sum of two eigenvectors of a matrix \(A\) corresponding to the same eigenvalue is always an eigenvector

5. Construct a matrix \(A\) with eigenvalues 1 and 3 and corresponding eigenvectors \((1, 2)^T\) and \((1, 1)^T\). Is such matrix unique?