Homework assignment, April 26, 2004.

Due Wednesday, 4/28 (collected)

1. Find singular value decomposition $A = W \Sigma V^*$ where $V$ and $W$ are unitary matrices for the following matrices
   a) $A = \begin{pmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{pmatrix}$
   b) $A = \begin{pmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{pmatrix}$

2. Find singular value decomposition of the matrix

   $A = \begin{pmatrix} 2 & 3 \\ 0 & 2 \end{pmatrix}$

   Find
   a) $\max_{\|x\| \leq 1} \|Ax\|$ and the vectors where the maximum is attained.
   b) $\min_{\|x\| \leq 1} \|Ax\|$ and the vectors where the minimum is attained.
   c) the image $A(B)$ of the closed unit ball in $\mathbb{R}^2$, $B = \{x \in \mathbb{R}^2 : \|x\| \leq 1\}$. Describe $A(B)$ geometrically.

3. True or false
   a) Singular values of a matrix are also eigenvalues of the matrix.
   b) Singular values of a matrix $A$ are eigenvalues of $A^*A$.
   c) If $s$ is a singular value of a matrix $A$ and $c$ is a scalar, then $|c|s$ is a singular value of $cA$.
   d) The singular values of any linear operator are non-negative.
   e) Singular values of a self-adjoint matrix coincide with its eigenvalues.

4. Let $A$ be an $m \times n$ matrix. Prove that non-zero eigenvalues of the matrices $A^*A$ and $AA^*$ (counting multiplicities) coincide.
   Can you say when zero eigenvalue of $A^*A$ and zero eigenvalue of $AA^*$ have the same multiplicity?
5. Let $s$ be the largest singular value of an operator $A$, and let $\lambda$ be the eigenvalue of $A$ with largest absolute value. Show that $|\lambda| \leq s$. 