Introduction to algebraic geometry
Brendan Hassett

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Errata in the first printing, corrected in the second printing

page 1, line 8:
Ending comma should be a period, i.e., change '$|\alpha| = \alpha_1 + \ldots + \alpha_n$,' to '$|\alpha| = \alpha_1 + \ldots + \alpha_n$.'

page 9, line 6: Exercise 1.7:
Change the first line 'Consider the morphism' to 'Let k be an infinite field. Consider the morphism'

page 10, line 3: Exercise 1.11.b:
'ϕ' should be 'ϕ^*'

page 32, line 11 Exercise 2.18.d:
Remove the following text: '*Hint:* Consider chains

$$m \supset m^2 \supset m^3 \ldots \supset p$$

where p is a nonmaximal prime and $m \supset p$ is maximal.'

page 49, line 9 from bottom:
The expression

$$k[W] = k[x_1, x_2, x_3, x_4, x_5]/\langle x_1(x_5 - 1), x_2(x_5 - 1), x_3x_5, x_4x_5\rangle$$

should read

$$k[W] = k[x_1, x_2, x_3, x_4, x_5]/\langle x_1(x_5 - 1), x_2(x_5 - 1), x_3x_5, x_4x_5, x_5(x_5 - 1)\rangle$$

page 75, line 5:

The typography of the displayed equation

$$\operatorname{Res}(f,g) = \det \begin{pmatrix} a_m & a_{m-1} & \dots & a_0 & 0 & 0 & \dots & 0 \\ 0 & a_m & a_{m-1} & \dots & a_0 & 0 & \dots & 0 \\ 0 & 0 & \ddots & \ddots & \dots & \vdots & \vdots & 0 \\ 0 & 0 & \dots & 0 & a_m & a_{m-1} & \dots & a_0 \\ b_n & b_{n-1} & \dots & b_0 & 0 & 0 & \dots & 0 \\ 0 & b_n & b_{n-1} & \dots & b_0 & 0 & \dots & 0 \\ 0 & 0 & \ddots & \ddots & \dots & \vdots & \vdots & 0 \\ 0 & 0 & \dots & 0 & b_n & b_{n-1} & \dots & b_0 \end{pmatrix}.$$

is confusing. It would be better to write:

$$\operatorname{Res}(f,g) = \det \begin{pmatrix} a_m & a_{m-1} & \dots & a_1 & a_0 & 0 & \dots & 0 \\ 0 & a_m & a_{m-1} & \dots & a_1 & a_0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & a_m & a_{m-1} & \dots & a_1 & a_0 \\ b_n & b_{n-1} & \dots & b_1 & b_0 & 0 & \dots & 0 \\ 0 & b_n & b_{n-1} & \dots & b_1 & b_0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & b_n & b_{n-1} & \dots & b_1 & b_0 \end{pmatrix}.$$

page 76, lines 6-7:
The expression

$$\delta_0(d)(A,B) = (r_{d-m},\dots,r_0,s_{d-n},\dots,s_0)\cdot$$

$$\overbrace{}^{m+1\text{ columns}} \quad \overbrace{}^{d-m\text{ columns}}$$

$$\begin{pmatrix} a_m & \dots & a_0 & 0 & 0 & 0 & \dots & 0 \\ 0 & a_m & \dots & a_0 & 0 & 0 & \dots & 0 \\ 0 & 0 & \ddots & \ddots & \dots & \vdots & \vdots & 0 \\ 0 & 0 & \dots & 0 & 0 & a_m & \dots & a_0 \\ b_n & \dots & \dots & b_0 & 0 & 0 & \dots & 0 \\ 0 & b_n & \dots & \dots & b_0 & 0 & \dots & 0 \\ 0 & 0 & \ddots & \ddots & \dots & \vdots & \vdots & 0 \\ 0 & 0 & \dots & 0 & b_n & \dots & \dots & b_0 \end{pmatrix} \begin{pmatrix} x^d \\ x^{d-1} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ x \\ 1 \end{pmatrix}.$$

$$\underbrace{}_{d-n\text{ columns}} \quad \underbrace{}_{n+1\text{ columns}}$$

is also confusing. It would be better to write

$$\delta_0(d)(A,B) \;=\; (r_{d-m},\ldots,r_0,s_{d-n},\ldots,s_0)\cdot \overbrace{}^{m+1\text{ columns}}\overbrace{}^{d-m\text{ columns}}$$

$$\begin{pmatrix} a_m & a_{m-1} & \ldots & a_1 & a_0 & 0 & \ldots & 0 \\ 0 & a_m & a_{m-1} & \ldots & a_1 & a_0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \ldots & 0 & a_m & a_{m-1} & \ldots & a_1 & a_0 \\ b_n & b_{n-1} & \ldots & b_1 & b_0 & 0 & \ldots & 0 \\ 0 & b_n & b_{n-1} & \ldots & b_1 & b_0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \ldots & 0 & b_n & b_{n-1} & \ldots & b_1 & b_0 \end{pmatrix} \begin{pmatrix} x^d \\ x^{d-1} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ x \\ 1 \end{pmatrix}.$$

$$\underbrace{}_{d-n\text{ columns}}\quad\underbrace{}_{n+1\text{ columns}}$$

page 79, line 8 from bottom:
The displayed formula

$$\begin{pmatrix} a_{m-1} & \ldots & a_0 & 0 & 0 & \ldots & 0 \\ 0 & a_{m-1} & \ldots & a_0 & 0 & \ldots & 0 \\ 0 & \ddots & \ddots & \ldots & \vdots & \vdots & 0 \\ 0 & \ldots & 0 & 0 & a_{m-1} & \ldots & a_0 \\ b_n & b_{n-1} & \ldots & b_0 & 0 & \ldots & 0 \\ 0 & \ddots & \ddots & \ldots & \vdots & \vdots & 0 \\ 0 & \ldots & 0 & b_n & b_{n-1} & \ldots & b_0 \end{pmatrix}$$

should read

$$\begin{pmatrix} a_{m-1} & \ldots & a_1 & a_0 & 0 & \ldots & 0 \\ 0 & a_{m-1} & \ldots & a_1 & a_0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \ldots & 0 & a_{m-1} & \ldots & a_1 & a_0 \\ b_n & b_{n-1} & \ldots & b_1 & b_0 & \ddots & \vdots \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ \ldots & 0 & b_n & b_{n-1} & \ldots & b_1 & b_0 \end{pmatrix}$$

page 114, line 11 from bottom, Exercise 7.17:
The expression

$$\pi : \mathbb{A}^n(k) \to \mathbb{A}^d(k)$$

should read

$$\pi : V \to \mathbb{A}^d(k)$$

page 117, lines 4-5 from bottom, Example 8.5:
'If $I = \langle xy, y^2\rangle$ ...' should read 'If $I = \langle xy, x^2\rangle$ ...'
The displayed equation

$$I = \langle y\rangle \cap \left\langle x^2, y\right\rangle$$

should read

$$I = \langle x\rangle \cap \left\langle x^2, y\right\rangle$$

page 152, line 6, Example 9.39:
The displayed matrix

$$\begin{pmatrix} z_{m\ 0} & z_{m-1\ 1} & \dots & z_{1\ m-1} \\ z_{m-1\ 1} & z_{m-2\ 2} & \dots & z_{0\ m} \end{pmatrix}.$$

is confusing and unattractive. It should be replaced by

$$\begin{pmatrix} z_m & z_{m-1} & \dots & z_1 \\ z_{m-1} & z_{m-2} & \dots & z_0 \end{pmatrix}.$$

page 192, lines 5-7, Example 11.19:
Add a period to the end of

$$\begin{aligned} (e_1 \wedge e_2 \quad + \quad & e_3 \wedge e_4)^2 = e_1 \wedge e_2 \wedge e_1 \wedge e_2 \\ & + e_1 \wedge e_2 \wedge e_3 \wedge e_4 + e_3 \wedge e_4 \wedge e_1 \wedge e_2 + e_3 \wedge e_4 \wedge e_3 \wedge e_4 \\ = \quad & 2e_1 \wedge e_2 \wedge e_3 \wedge e_4 \end{aligned}$$

so it reads

$$\begin{aligned} (e_1 \wedge e_2 \quad + \quad & e_3 \wedge e_4)^2 = e_1 \wedge e_2 \wedge e_1 \wedge e_2 \\ & + e_1 \wedge e_2 \wedge e_3 \wedge e_4 + e_3 \wedge e_4 \wedge e_1 \wedge e_2 + e_3 \wedge e_4 \wedge e_3 \wedge e_4 \\ = \quad & 2e_1 \wedge e_2 \wedge e_3 \wedge e_4. \end{aligned}$$

page 199, line 5, Example 11.31
Delete the comma immediately following the equation

$$\omega \wedge \omega = 0,$$

so it reads

$$\omega \wedge \omega = 0$$

page 242, line 10, (Theorem A.14 Sketch Proof)
Add a proof completion sign '□' at the end of the line, i.e., after 'content 1 in $R[x]$.'

Errata to be corrected in subsequent printings

page xi, line 4 from bottom
'Macaulay II' should read 'Macaulay2'

page 6, line 12
'do not affect the number conditions imposed' should be 'do not affect the number of conditions imposed'

page 32, line 17, (Exercise 2.18(e))
This exercise requires the Nullstellensatz and thus should be postponed to later in the book.

page 44, line 6
'$V(\{g_1, \dots, g_m\})$' should read '$V(\{g_1 \cdots g_m\})$'

page 46, Definition 3.45
'rational maps $\rho' : \mathbb{A}^n(k) \dashrightarrow W$ admissible on V.' should read 'rational maps $\rho' : \mathbb{A}^n(k) \dashrightarrow \mathbb{A}^m(k)$ admissible on V such that $\rho'(V) \subset W$.'

page 58, line 6
'elmination' should be 'elimination'

page 62, line 9
font size of $\frac{1}{x}$ is incorrect

page 72, line 8, Exercise 4.13
The last term 'Scroll$(\mathbb{A}^3; \phi(1), \phi(2)$' misses a right parenthesis.

page 92, line 10 from bottom
'$V(f)$ is irreducible if and only if f is irreducible.' should read '$V(f)$ is irreducible if and only if f is a power of an irreducible polynomial.'

page 92, line 1 from bottom and page 93, line 1
I_Z should read $I(Z)$ and I_V should read $I(V)$

page 121, line 6 from the bottom
In the expression '$E\det(M) = (AE - sBC)\ldots$' the '$s$' is a typo and redundant.

page 123, line 2 to 3
The statement 'the nilpotents in R/Q are images of the associated prime $P = \sqrt{Q}$' should read 'the zero divisors in R/Q are images of the associated prime $P = \sqrt{Q}$'

page 128, line 4 from bottom
The definition of indeterminacy ideal is incorrect. The display

$$\{r \in k[V] : r\rho^*f \in k[V] \text{ for each } f \in k[W]\}$$

should read

$$\{r \in k[V] : \text{ for each } f \in k[W], r^m\rho^*f \in k[V] \text{ for some } m \in \mathbb{N}\}$$

page 129, line 8
'$g\rho^*k[W]$' should be replaced by '$g^M\rho^*k[W]$ for some M' given the change in the definition of indeterminacy locus

page 130, last 10 lines on computing indeterminacy
This must be revised given the change in the definition of the indeterminacy ideal. The algorithm computes an ideal whose radical is the indeterminacy ideal.

page 145, line 6
'If $H \in k[x_0, \ldots, x_n]$ is homogeneous' should read 'If $H \in k[x_0, \ldots, x_n]$ is homogeneous of positive degree'

page 172, line 14
'$x_0^N h \in F$' should read '$x_0^N h = F \in J$'

page 177
We claim that for a projective variety X and a morphism ϕ, $\mathrm{cone}(\phi(X))$

equals $\phi(\text{cone}(X))$. After taking Zariski-closures they become equal, over an infinite field. Without passing to the Zariski closure, some condition on the underlying field is needed for this to be true, e.g., if the polynomials defining ϕ have degree d then the field needs to have d-th roots. For example, over the real numbers, if $X = [1 : 1]$ and $\phi(x, y) = [x^2 : y^2]$, then the cone over $\phi(X)$ is the line $x = y$ in $\mathbb{R}^2$, but $\phi(\text{cone}(X))$ is only the nonnegative ray within this line. (Thanks to C. Vinzant for pointing this out.)

page 188, line 8, (Lemma 11.14)
'For each $N \times (M - N)$ matrix' should read 'For each $M \times (N - M)$ matrix'

page 190, line 9 from bottom
'These we have a natural identification' should read 'These have a natural identification'

page 214, line 11
'$k[x_0, \ldots, x_n/J$' should read '$k[x_0, \ldots, x_n]/J$'

page 220, line 3
'$I(U_0 \cap X(J)$' should read '$I(U_0 \cap X(J))$'

page 220, line 10
'W_t is unbounded' should read '$\dim(W_t)$ is unbounded'

page 240, line 19
'and g is a multiple of I' should read 'and g is a multiple of f'