ARITHMETIC OF K3 SURFACES OF DEGREE TWO

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Abstract.

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1. Outline

Let (S, f) be a polarized complex K3 surface of degree two, realized as a double cover

$$\pi : S \to \mathbb{P}^2$$

branched over a sextic plane curve B. The intersection form on $H^2(S, \mathbb{Z})$ is denoted $(-,-)$. Let Γ denote the orientation-preserving automorphisms of $H^2(S, \mathbb{Z})$ respecting $(-,-)$ and fixing the polarization f.

Consider the subgroup

$$H^2(S, \mu_2)_0 := f^\perp \subset H^2(S, \mu_2)$$

and the subquotient

$$H = H^2(S, \mu_2)_0 / \langle f \rangle ,$$

which is isomorphic to $(\mathbb{Z}/2\mathbb{Z})^{20}$. We have a natural isomorphism [**?**, pp. 587]

$$H \xrightarrow{\sim} H^1(C, \mathbb{Z}/2\mathbb{Z}).$$

Proposition 1. *H has three orbits under the action of Γ*

- 0
- $\beta \neq 0 \in H$ *with* $(\beta, \beta) = 0$*;*
- $\beta \in H$ *with* $(\beta, \beta) = 1$.

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Let $\mathrm{Br}(S)[2]$ denote the two-torsion elements of the Brauer group of S and $\mathrm{Br}(S)[2]_0$ those elements orthogonal to f. Thus we have a surjection

$$H^1(C, \mathbb{Z}/2\mathbb{Z}) \simeq H \twoheadrightarrow \mathrm{Br}(S)[2]_0.$$

Let $\mathcal{K}_{2g-2}$ denote the moduli space of polarized K3 surfaces of degree $2g-2$.

Proposition 2. *There exists a morphism*

$$\varphi : \mathcal{K}_{8g-8} \to \mathcal{K}_{2g-2}$$

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