Final Exam
Math 102 – Spring 2008

Instructions: This is a 3 hour exam. You may not consult any notes or books during the exam, and no calculators are allowed. Show all of your work on each problem. Attach extra paper if you need more space.

Name:

Instructor:

Honor Pledge: On my honor, I have neither received nor given any unauthorized aid on this exam.

Signature:

Problem	Score
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
Total	

1. **Problem 1 (10 points)**
   Evaluate the integral $\displaystyle\int \frac{x^2+4}{x^3+2x}\,dx$.

2. **Problem 2 (10 pts)**
   Evaluate the integral $\displaystyle\int \tan^3\theta \sec^4\theta\,d\theta$.

3. **Problem 3 (10 points)**
Determine whether or not the improper integral converges. If it converges, find its value. If it diverges to $\pm\infty$, specify which one.

$$\int_0^9 \frac{1}{\sqrt[3]{x-1}}\,dx$$

4. **Problem 4 (10 points)**
Evaluate the integral $\displaystyle\int \frac{x^2}{\sqrt{1-x^2}}\,dx$. Express your final answer in terms of x.

5. **Problem 5 (5 points)**
Use the Taylor series for $\sin x$ to find a power series representation of $\frac{\sin(-x^2)}{x}$.

6. **Problem 6 (10 points)**
Consider the series $\sum_{n=2}^{\infty} \frac{(-1)^n \ln n}{n}$.

(a) Does the series converge? Verify all hypotheses for any tests used.

(b) Does the series converge absolutely?

7. **Problem 7 (10 pts)**

Does the series $\sum_{n=2}^{\infty} \frac{(-2)^n}{3^{n-1}}$ converge or diverge? If it converges, find its sum.

8. **Problem 8 (10 pts)**

Find the interval of convergence for the power series $\sum_{n=1}^{\infty} \frac{(-1)^n (x-1)^n}{2n+1}$. Don't forget to check the endpoints.

9. **Problem 9 (5 pts)**

What does it mean for the infinite series $\sum_{n=1}^{\infty} a_n$ to converge to a number L?

10. **Problem 10 (15 points)**
Consider the polar equations $r = 1$ and $r = 2 + 2\cos\theta$.

(a) Sketch the graphs of both given polar curves on the same plot.

(b) Find the values of θ where the given curves intersect.

(c) Find the area between the given curves (outside $r = 1$ and inside $r = 2 + 2\cos\theta$).

11. **Problem 11 (5 points)**

Find the equation of the line tangent to the parametric curve $x = t^2 - t$, $y = t^3 + 1$ at $t = 2$.

12. **Problem 12 (10 points)**
Find the arclength of the parametric curve $x = t^3 + 1$, $y = t^2$ over the interval $0 \leq t \leq 1$.

13. **Problem 13 (10 points)**
Find the area between the x-axis and the parametric curve $x = e^t$, $y = t^2$ for $0 \leq t \leq 2$.

Half and double angle identities

$$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$$

$$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

Integrals of basic trigonometric functions

$$\int \tan x\,dx = -\ln|\cos x| + C = \ln|\sec x| + C$$

$$\int \cot x\,dx = \ln|\sin x| + C = -\ln|csc x| + C$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x\,dx = -\ln|\csc x + \cot x| + C$$