Dan Abramovich - Sabbatical report

June 14, 2022

1. **Nontechnical introduction.** A sabbatical during a pandemic is very different from a usual sabbatical: one cannot fully focus on renewing one's own research when one is involved in the careers of young people, as I have been. I suspect many sabbatical reports during this pandemic are a bit unusual, and this is no exception: I spent much of the sabbatical, taken during calendar year 2021, working with undergraduate students, graduate students and postdoctoral fellows on their projects.

Since we are in the business of doing something unusual, I now try to explain one aspect of my work in nontechnical terms.

1.1. *Singularities and their resolution.* One aspect of my research is about singularities. Raise your eyes and look at the corner of your office, say the southwest corner. Most likely you see two planes — the walls — meeting along a line - the corner of your room. This is an example of a singularity. While the walls are nice and smooth, the corner is different: an ant walking at steady pace along one wall towards the other would have to turn sharply and suddenly upon meeting the corner.

This corner singularity appears in almost all subjects of mathematics, and in particular in my subject, *algebraic geometry*, where geometry is determined by algebraic equations. You can put coordinates for points in your room, where one wall is defined by $x = 0$ and the other by $y = 0$. Their union is determined by the algebraic equation $xy = 0$, while the singularity, the corner, is their intersection $x = y = 0$, where only the height z is varying.

This is also a good example for resolving a singularity. A resolution of a singularity is a process where you take a singular object, in this case the boundary of your office described by $xy = 0$, and re-parametrize it by something nice and smooth. Remembering that the boundary of your room was put together by combining two walls, it is resolved by taking the two walls apart and laying them, so to speak, side by side.

Singularities are in general much more complicated than a corner, and their resolution is therefore much more technical. The idea is however the same: take a singular object, and re-parametrize it by something nice and smooth.

2. **Combinatorial algebraic geometry.** Now the technical report.

During Spring 2021 I participated in the ICERM program *Combinatorial Algebraic Geometry*. The program was moved online due to the pandemic.

I participated in the program's seminars, as well as workshops online. As one outcome I formed a collaboration with Lars Kastner on computer implementation, and together we led a group of undergraduates who implemented [1] algorithms A through D of Bergh's destackification algorithm [9]. One discovery is that Bergh's algorithm is highly inefficient, and discussions with Rydh seem to lead to a much more efficient algorithm.

In addition I organized a focused seminar as part of the program, on the work [7] of Adiprasito Liu and Temkin. We went in detail through the central combinatorial arguments of the paper and provided the authors with a long list of comments.

I worked with Netanel Friedenberg as his program mentor. We attempted to attack the problem of polystable reduction, where we met obstructions to several approaches we attempted.

3. **Log smooth reduction and resolution of singularities.** During July and November-December 2021 I visited my collaborator Temkin in Jerusalem. In addition I spent September-October 2021 at the Mittag-Leffler institute, interacting in person with many visiting researchers.

Temkin and I explored a number of new directions, while mostly our work focused on revising the manuscript [6] with a fully functorial log smooth reduction result. A whole lot of progess was made, though the final manuscript is not yet complete.

Two other collaborations in this general direction were largely completed:

- with Bernd Schober, we revisited resolution of singularities of a confguration of binomial hypersurfaces in affine space [3], providing
  - a highly computable explicit and efficient one-step simultaneous resolution method using toric stacks.
  - A simple normal crossings resolution of the configuration in every non-exceptional characteristic, and
  - A resolution of a binomial ideal in any characteristic.
- With Ming Hao Quek we upgraded the weighted resolution algorithm in the logarithmic case to a normal-crossings resolution, using caninical Artin stacks, [2]. In particular we provide such a normal-crossings principalization algorithm, a normal-crossings embedded resolution, and a normal-crossings non-embedded resolution in characteristic 0.

A collaboration, largely with Quek and Schober, on positive characteristic resolution of surfaces using weighted resolution is in progress.

I continued to advise my PhD student Quek, who embarked on an ambitious project around the monodromy conjecture for motivic zeta functions of Newton-nondegenerate hypersurfaces.

During October 25-29, 2021 I co-organized an oberwolfach seminar on new techniques in resolution of singularities, with Anne Frühbis-Krüger, Michael Temkin and Jarosław Włodarczyk. This will lead to a book publication, including several chapters already written.

3.1. *The geometry of weighted blowings up.* Our work [5, 4] showed that stack theoretic weighted blowings up should be just as useful as classical blowings up. During the sabbatical I spend a considerable effort outlining a plan for the computation of various invariants of weighted blowings up. One aspect where the plan is complete, and being carried out by two of my students — Veronica Arena and Stephen Obinna — is the complete computation of the *integral* Chow rings of weighted projective stack bundles and of smooth weighted blowings up. The first step is already written in a preprint [8].

In short, Let X be a smooth variety, $N \to X$ a weighted affine bundle with $\mathbb{G}_m$-equivariant Chern class $P(T)$, and $\mathbb{P} \to X$ the resulting weighted projective stack bundle. Then $A^*(\mathbb{P}) = A^*(X)[T]/(P(T))$.

A similarly nice formula is given for the blowup, but its proof, which is in the writing, is quite a bit more demanding.

4. **Arithmetic.** During this year I continued to advise my PhD student Tangli Ge. He produced a very strong thesis and graduated in 2022. We worked hard together on a project related to his thesis, but have no result to report on that.

5. **Postdoctoral advising.** During this year I continued to meet regularly with Jeremy Usatine, a Tamarkin assistant professor at Brown, who had significant achievements during this period. In addition I met regularly with Bernd Schober, a postdoc in Germany with whom I developed a project, and with Hülya Argüz, a postdoc in France who was planning to come to Brown.

6. **Lectures.**

- Moduli and Hodge Theory, IMSA, Miami (online), *Semistable reduction - a progress report*, February 1, 2021

- Geometry and Analysis Seminar, Oxford (online), *Punctured invariants and gluing*, February 8, 2021.
- Singularities in positive characteristics, CIRM, Luminy, France, *Stacks in the service of resolution*, July 16, 2021.
- Oberwolfach Seminar: New Techniques in Resolution of Singularities, MFO, Germany.
  Title: Stacks, weighted resolution, and logarithmic resolution. Dates: October 25-29, 2021.
- Einstein Institute of Mathematics, Hebrew University, Jerusalem, Israel.
  Title: Logarithmic Geometry and Moduli Spaces. Dates: November-December, 2021.
- Real and Complex Geometry, Tel-Aviv University (online), *Punctured logarithmic maps*, December 16, 2021.
- Colloquium, Tel-Aviv University, *Resolving singularities in families*, December 20, 2021.
- Algebra Seminar, Technion, Israel, *Resolution of singularities using weighted and logarithmic blowing up*, December 23, 2021.

In addition to these events I attended two workshops at the ICERM program and two at the Mittag-Leffler program.

7. **Nontechnical conclusion.** I feel privileged to have had this sabbatical. The list of achievement above gives me a good feeling. But the salient feeling I had at the end of my sabbatical, and still have now, is the same as everybody's: I am depleted, demoralized, and worried about everyone around me.

References

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