

Math 2710V Final Exam:

Instructions: Please complete all the problems. Note, however, that you are allowed to work together. Collectively you may submit just one answer to each question. The exam is due by the end of the ICERM semester program. Show your work.

- 1: Build a battery-powered, robotic arm which does the belt trick in such a way that the terminal object spins in place without any vertical motion.
- 2: Fabricate a Coke bottle whose cross sections are Julia sets of quadratic polynomial maps indexed by a continuous path in the Mandelbrot set.
- 3: Print out a beer coaster illustrating a convex projective triangle group.
- 4: Make a woodcut showing the hierarchical structure of the Penrose tilings.
- 5: Make a woodcut showing the disks in an integral Apollonian packing.
- 6: Fabricate the metric balls of radius k , for $k = 1, 2, 3, 4, 5$, in the solvable lie group Sol.
- 7: Implement ray tracing in one of the following two Thurston geometries:
 - Nil
 - Sol
- 8: Fabricate an isometrically embedded patch of the hyperbolic plane. Then use this model to construct hyperbolic paper.
- 9: Recall that quaternions can be used to construct a double cover from $S^3 \times S^3$ to $SO(4)$, incorporate this map into your virtual reality program.
- 10: Make a demonstration of billiards in a rectangle which involves mirrors and a laser pointer.