Here are some hyperbolic manifold questions:

1: Let $g \geq 2$ be an arbitrary integer. Let P_g be the (solid) geodesic regular hyperbolic polygon having $4g$ sides and interior angles $\pi/(2g)$. Let Σ_g be the quotient space obtained by isometrically gluing the opposite sides of P_g in the orientation preserving way. (As you move leftward across the top edge, the corresponding point moves leftward across the bottom edge, etc.) Show that Σ_g is a hyperbolic surface of genus g. The idea is to construct local coordinate charts into $\boldsymbol{H}^2$ around each point and check that the overlap functions are restrictions of hyperbolic isometries.

2: Prove that there exists a regular hyperbolic dodecahedron whose dihedral angles are all $2\pi/5$, but no regular hyperbolic dodecahedron having dihedral angles $\pi/7$. What is the cutoff?

3: Let D_5 be the hyperbolic dodecahedron from Problem 3. Find the completely symmetric way of gluing the opposite sides of D_5 so that the resulting space is a hyperbolic 3-manifold. This is what I tried to do in class. How many different such gluings are there?

4: Let O_4 denote the regular ideal octahedron. Prove that the dihedral angles of O_4 are all $\pi/4$, and find a way to glue the opposite faces of O_4, in a completely symmetric way, so that the resulting space is a hyperbolic 3-manifold. It turns out that the manifold you get is <u>not</u> the Whitehead link complement, but it is still a nice manifold. The Whitehead link complement comes from a different gluing pattern on the faces of O_4.

5: Prove that there exists a regular spherical dodecahedron D_3 whose dihedral angles are all $2\pi/3$. This means that the faces of D_3 are all subsets of great spheres in the 3-sphere.

6: Prove that there exists a completely symmetric and isometric gluing of the sides of D_3 so that the resulting quotient is a compact 3-manifold that is locally isometric to the 3-sphere. This is the other completely symmetric gluing of the regular dodecahedron. Incidentally, I think that I got the two of these mixed up in class. But if you do this exercise and also Exercise 3 you will have it straight.

7: Prove that there exists a gluing of the faces of two (disjoint) regular ideal tetrahdra so that the resuting quotient is a hyperbolic 3-manifold. You want there to be 6 edges glued together. This is probably the most famous of all hyperbolic 3-manifolds, and you can find it in Bill Thurston's *Notes*.

8: Prove that for each triangle τ there is an ideal tetrahdron whose dihedral angles are the same as the angles of τ It will turn out that the opposite edges of the ideal tetrahdron will have the same dihedral angle, and each angle of τ will arise as dihedral angles for a pair of opposite edges of the tetrahedron.

9: Examine your gluing from Problem 7 and convince yourself that you can change the shapes of the pair of tetrahedra so that the resulting space you get (with the same gluing pattern) is still a hyperbolic 3-manifold. The manifolds you get this way are generally not globally isometric to the one from Problem 7. Also, they will generally not be *complete*. There will be geodesics in these manifolds which are only defined for a finite amount of time. This weird property is the beginning of Thurston's Dehn surgery theorem.

10: Ask me about Thurston's Dehn surgery theorem and I will try to explain what I know about it.