HW Assignment 2:

1: Let G be any finite graph. Prove that one can embed K in space so that the edges are straight line segment. (This is the proof-practice problem.)

2: Suppose that $A = \{A_1, ..., A_n\}$ is a list of distinct points in the plane and $B = \{B_1, ..., B_n\}$ is a list of distinct points in the plane and $A \cap B = \emptyset$. Prove that there are disjoint embedded polygonal paths $P_1, ..., P_n$ such that A_i and B_i are the endpoints of P_i for each $i = 1, ..., n$.

3: With the same set-up as in Problem 2, prove that there exists some permutation σ so that one can connect A_i to $B_{\sigma(i)}$ by a line segment so that all line segments are disjoint from each other. In other words, if you are allowed to choose which points are connected to which, you can use line segments.

4: Prove that $K_{4,4}$ and K_7 can each be embedded in the torus.

5: Prove by induction on the number of faces that a planar graph is bipartite if and only if every face has an even number of sides. This is problem 6.1.20 in West's book.

6: Let T be a tree embedded in the plane using straight line segments. Prove that $\boldsymbol{R}^2 - T$ has one face. That is, every two points in $\boldsymbol{R}^2 - T$ can be joined by a polygonal path that avoids T. After doing this, give an argument which uses induction on the number of cycles, to prove Euler's formula for connected planar polygonal graphs. This is problem 6.1.24 in West's book.

7. Let C be a circle. Suppose that m chords of C are drawn so that no three share a common point and no two share an endpoint.Let p be the number of pairs of chords that cross. In terms of m and p compute the number of segments and the number of regions formed inside C. This is essentially problem 6.1.28 in West's book.

8: Construct, for each even $n > 8$, an example of a planar graph which has exactly 4 vertices of degree less than 6. This is part of Problem 6.1.35 in West's book. (For half credit, construct such graphs for infinitely many n.)