HW Assignment 3:

1: Find a proper 4-coloring of the vertices of the icosahedron.

2: How does the 5-color theorem work on the torus? Try to imitate the proof in class and see how far you get.

3: Suppose that every point in the plane is colored one of 3 colors. Prove that there must be 2 points, exactly one unit apart, which have the same color. (This is part of problem 5.1.25)

4: Prove that it is possible to color every point of the plane with one of 7 colors, so that no two points which are 1 unit apart have the same color. (This is also part of problem 5.1.25)

5: This is Problem 3.1.20 in the book: Suppose that G is a graph such that every pair of odd cycles in G intersect. Prove that G is 5-colorable. (Hint: First consider the case when G has a single odd cycle.)

6: This is problem 3.1.22 from the book: Suppose that you draw a finite number of lines in the plane, such that no three contain the same point. Form a graph whose vertices are intersection points and whose edges join consecutive points on the same line. Prove that this graph is 3-colorable.

7: This is problem 5.3.4: Prove that $\chi(C_n, k) = (k-1)^n + (-1)^n(k-1)$.

8: This is Problem 5.3.5. Let G_n be the same graph considered in HW set 1. Figure 1 shows it.

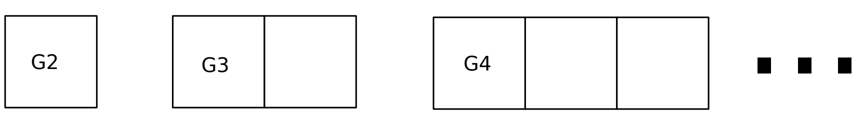

Figure 2: The graphs $G_1, G_2, G_3, \ldots$.

Prove that $\chi(G_n, k) = (k^2 - 3k + 3)^{n-1}k(k-1)$. (Hint: Look at Example 5.3.7.) Note: In the previous HW assignment, I had the index n shifted by 1, so that my G_n was West's G_{n+1}. This wasn't important for the previous problem, but it is important here.