HW Assignment 4:

1: This is problem 3.1.11 in the book: Let C and C' be cycles in a graph. Prove that $C\Delta C'$ (the union of edges either in C or C' but not in both) decomposes into cycles.

2: Let G be the Peterson graph. Say that a *flag* of G is a pair (v, C) where C is a 5-cycle in G and v is a vertex of C. Prove that for any two flags (p_1, C_1) and (p_2, C_2) there is a symmetry of G – i.e. a graph automorphism – which maps (p_1, C_1) to (p_2, C_2).

3: Recall that the Peterson graph is the quotient of the dodecahedron graph by the anti-podal map. Describe the perfect matchings of the Peterson graph in terms of cycles on the dodecahedron graph. (Hint: problem 2 will help.)

4: This is problem 3.1.31. Use the Konig-Egervary theorem to deduce Hall's Theorem.

5: Is it possible to cover a $(2n) \times (2n)$ checkerboard with non-overlapping dominoes in such a way that only the top-left square and the bottom-right square are uncovered?

6: Let T_1 be the tiling of the plane by unit squares whose vertices have integer coordinates. Let T_2 be the result of rotating T_1 about the origin by some angle θ. Prove that it is possible to find a bijection between the squares of T_1 and the squares of T_2 in such a way that the matched squares are within 10 units of each other. The matching will depend on θ. Hint: try to make this look like Hall's Matching Theorem.

7: This is problem 3.3.6 in the book: Prove that a tree has a perfect matching if and only if for every $v \in T$ the graph $T - v$ has exactly one connected component with an odd number of vertices.

8: This is problem 3.2.14 in the book, expressed in neutral language. Prove that in the proposal algorithm, with the ants proposing to the bees, that no ant is rejected by every bee.