HW Assignment 7:

1: Deduce Hall's Theorem from the Konig-Evergary Theorem.

2: Is it possible to cover a $(2n) \times (2n)$ checkerboard with non-overlapping dominoes in such a way that only the top-left square and the bottom-right square are uncovered?

3: Let T_1 be the tiling of the plane by unit squares whose vertices have integer coordinates. Let T_2 be the result of rotating T_1 about the origin by some angle θ. Prove that it is possible to find a bijection between the squares of T_1 and the squares of T_2 in such a way that the matched squares are within 10 units of each other. The matching will depend on θ. Hint: try to make this look like Hall's Matching Theorem.

4: This is problem 3.3.6 in the book: Prove that a tree has a perfect matching if and only if for every $v \in T$ the graph $T - v$ has exactly one connected component with an odd number of vertices.