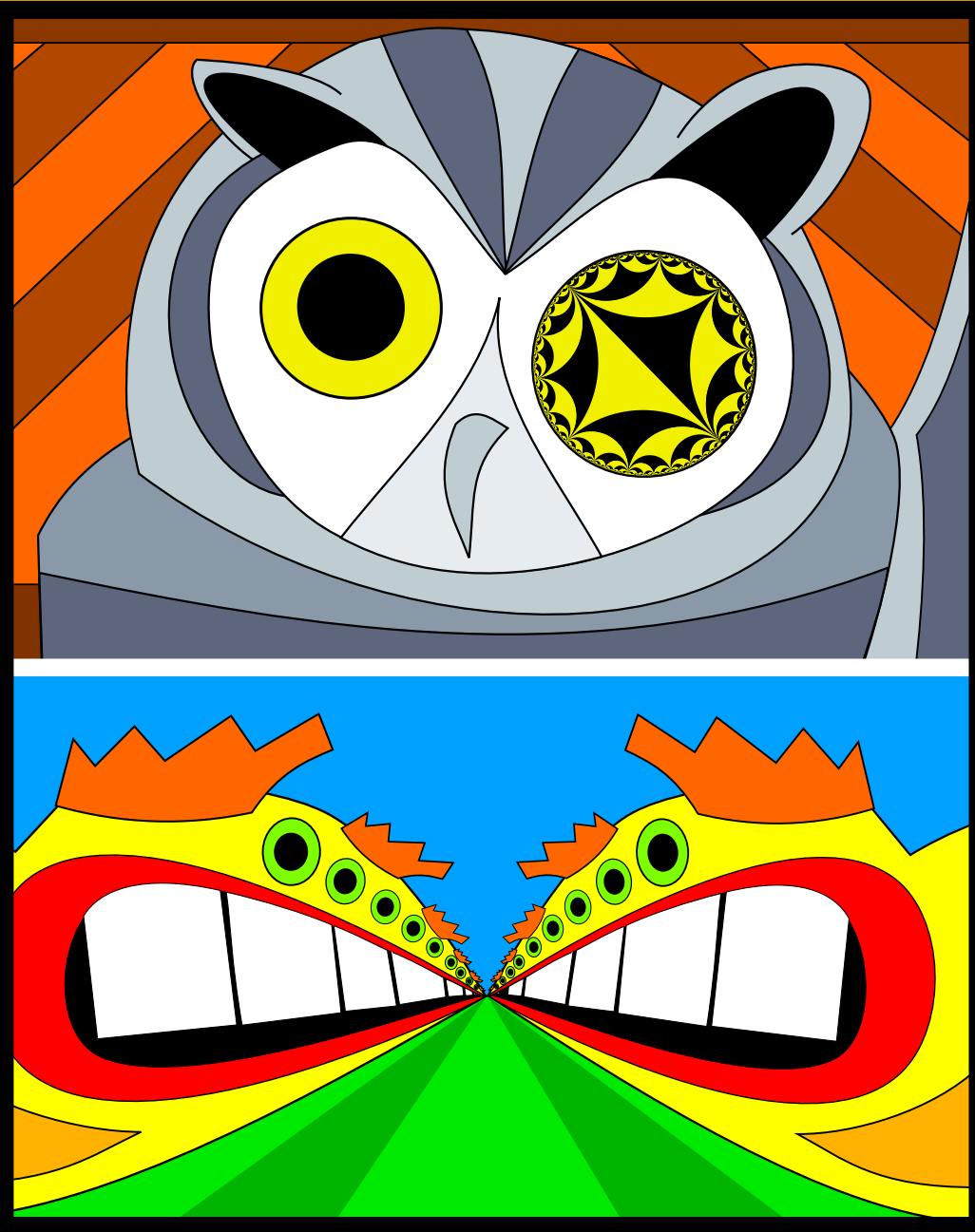


# Life on the Infinite Farm

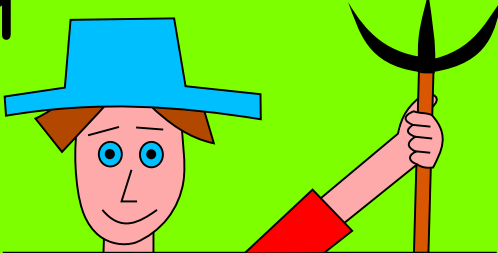
Richard Evan Schwartz



**To all my math teachers.**


## Note to the reader

1



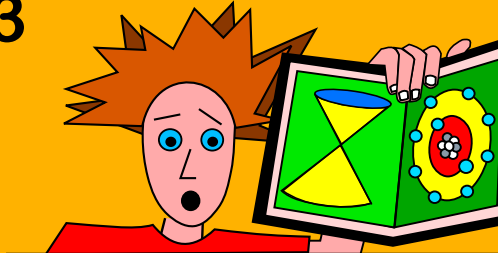
When I started writing this book, I planned to make it simple enough so that anyone old enough to read could enjoy it.

2



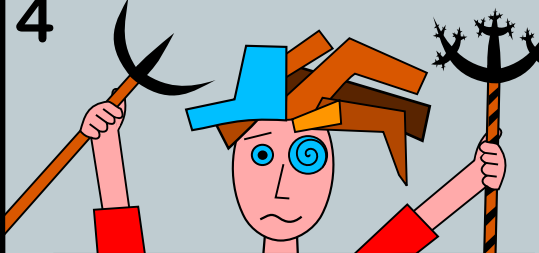
But then I went a little crazy and put in all kinds of fancy ideas that I found interesting.

3



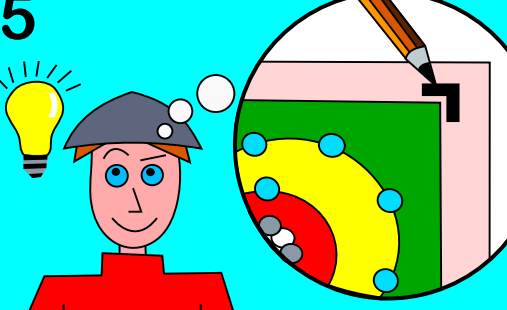
When I looked at the pages that mention atoms and special relativity, I wondered if I had gone too far.

4



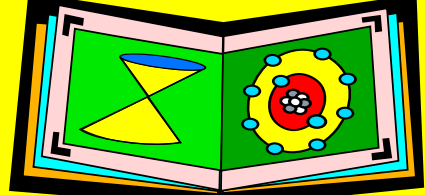
I ended up making both a simple version and an advanced version, but I couldn't decide which one to show you.

5



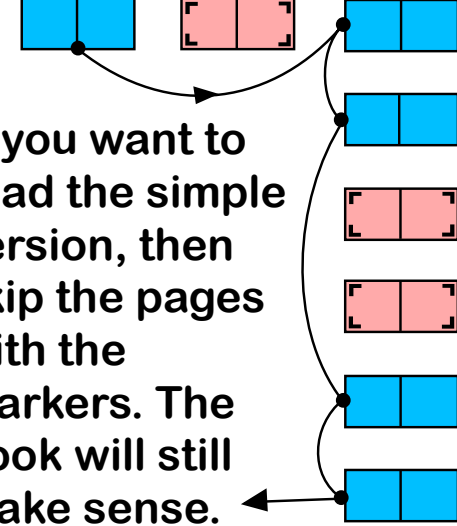
Then I had an idea about how to show you both versions at the same time.

6



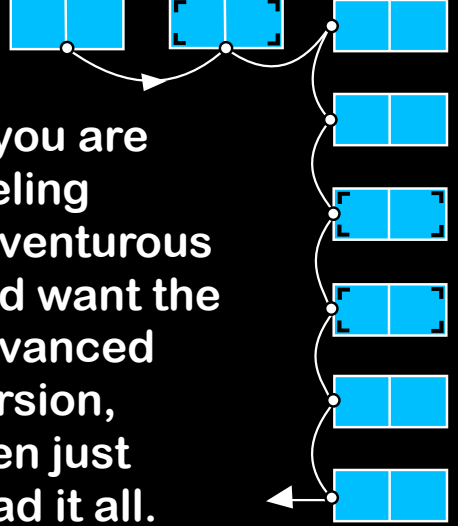
This book has all the pages, simple and advanced, but I put little black markers in the corners of all the advanced ones.

7



If you want to read the simple version, then skip the pages with the markers. The book will still make sense.

8

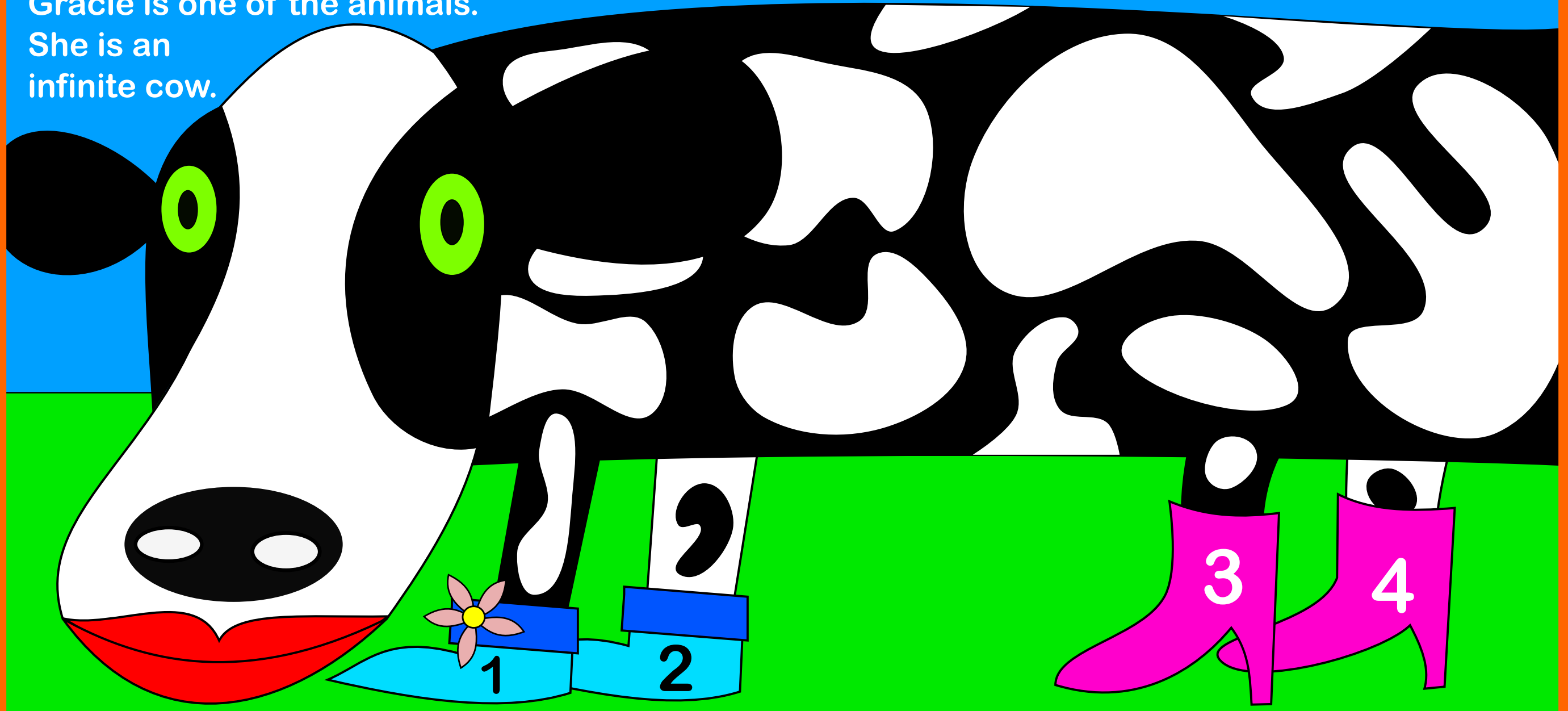


If you are feeling adventurous and want the advanced version, then just read it all.

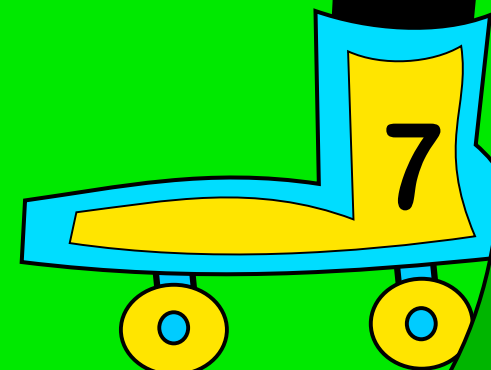
Who are the animals  
on the infinite farm?



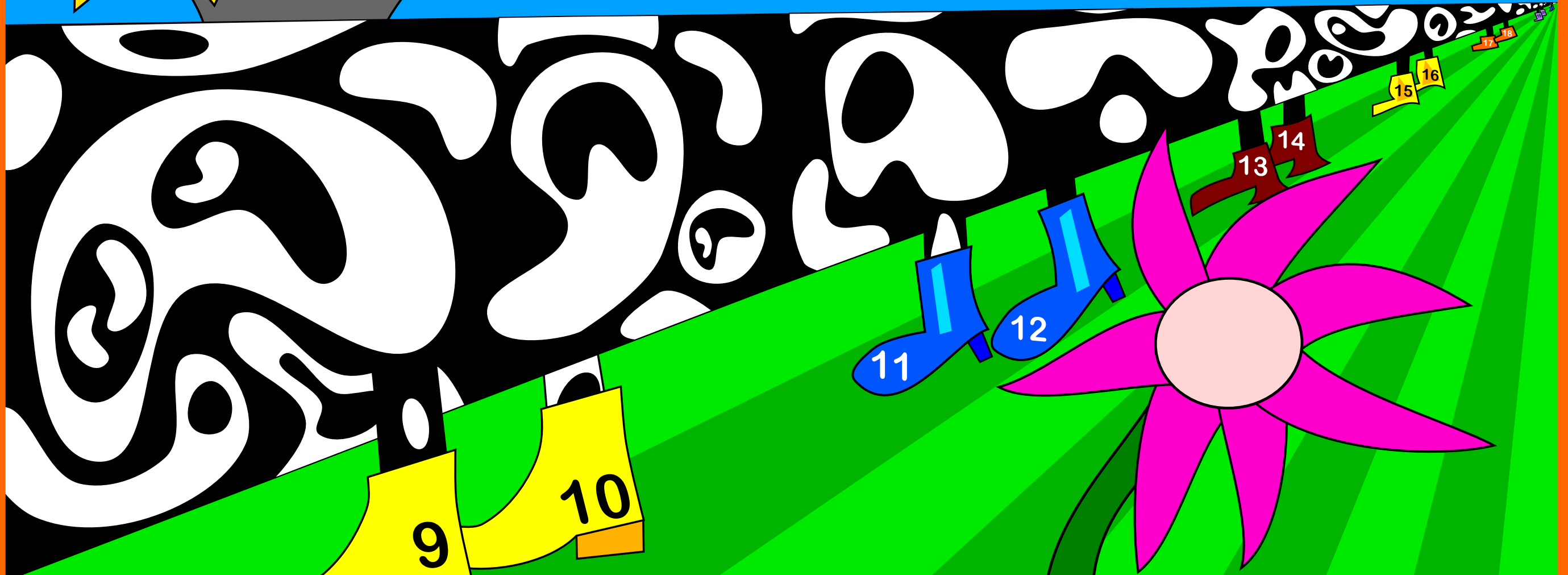
Gracie is one of the animals.  
She is an  
infinite cow.

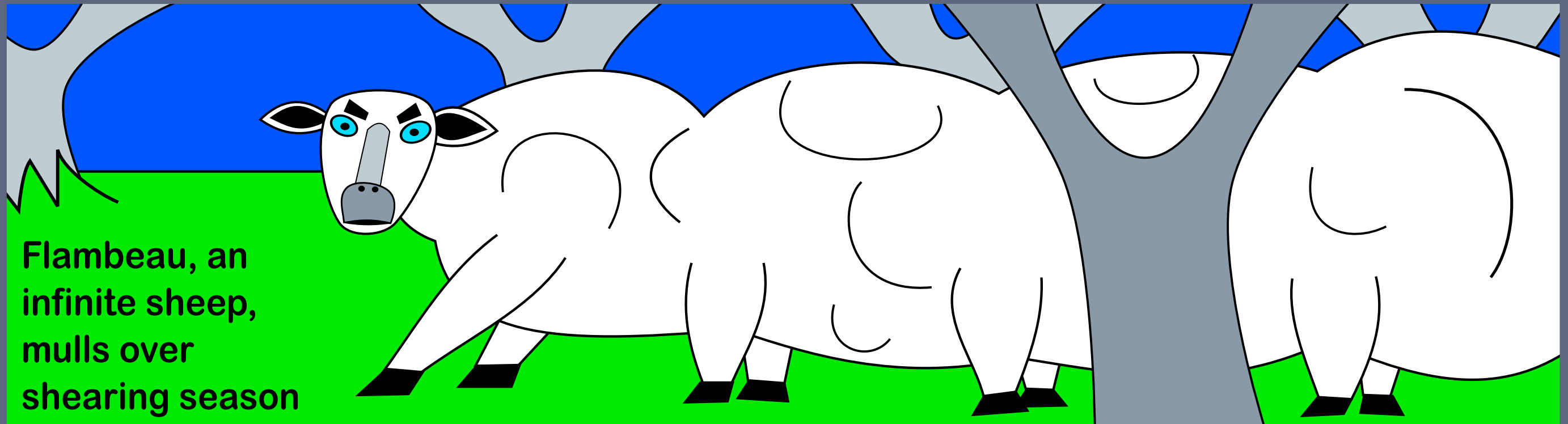


Gracie  
loves  
shoes.  
She has  
one  
on every  
foot ...

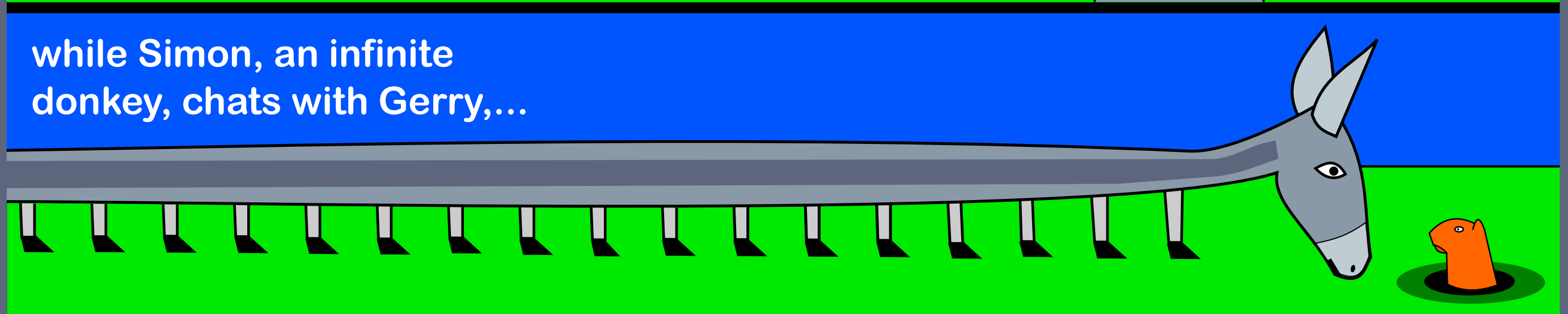


and her feet  
go on forever!





Flambeau, an  
infinite sheep,  
mulls over  
shearing season



while Simon, an infinite  
donkey, chats with Gerry,...

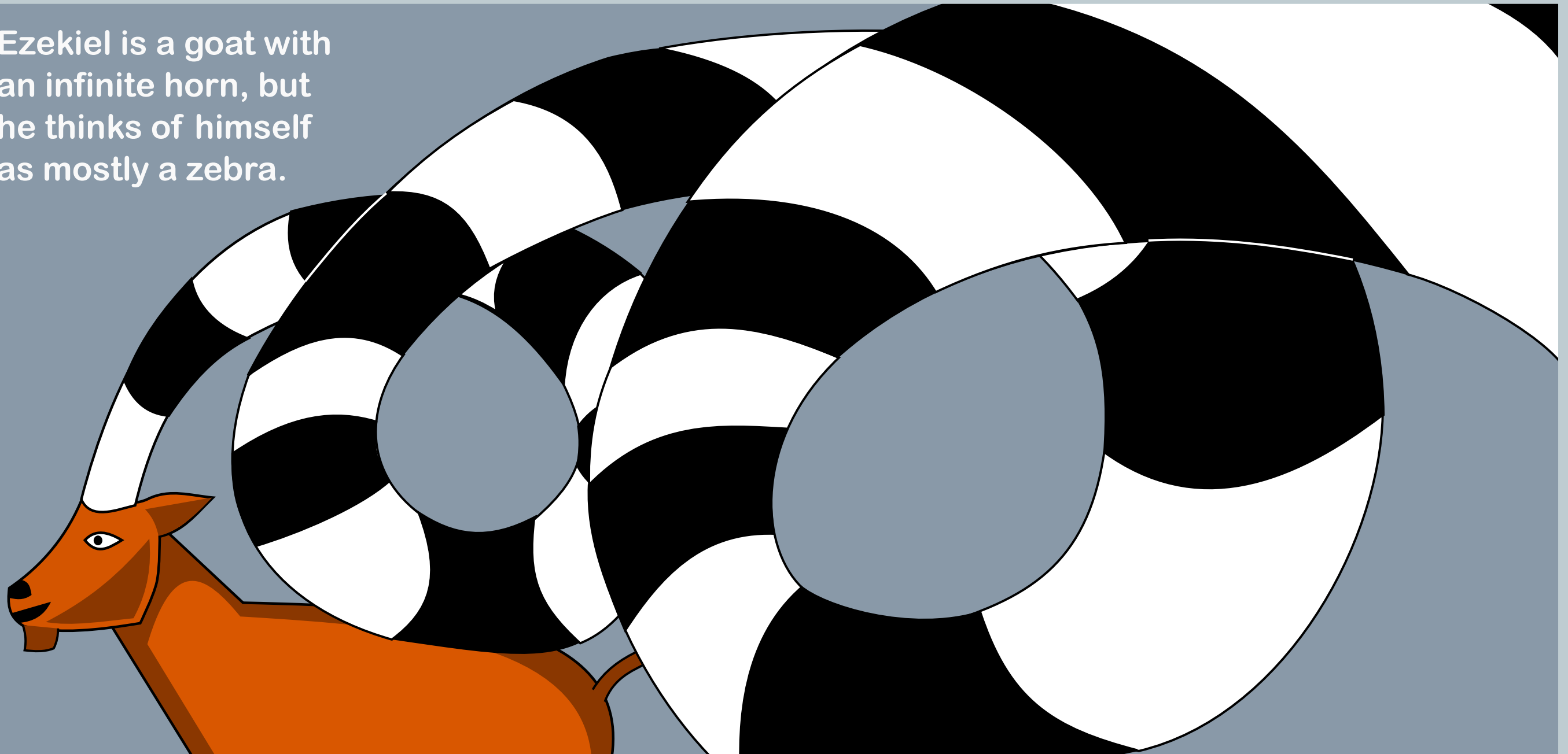




an infinite gopher.



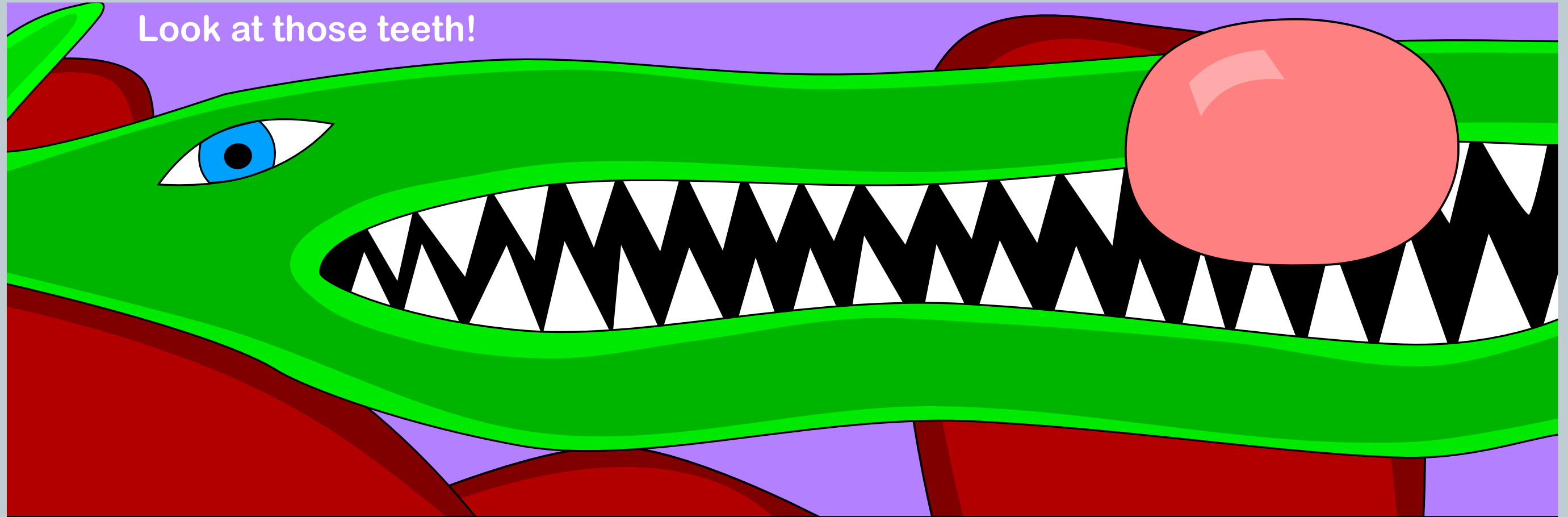
Ezekiel is a goat with  
an infinite horn, but  
he thinks of himself  
as mostly a zebra.



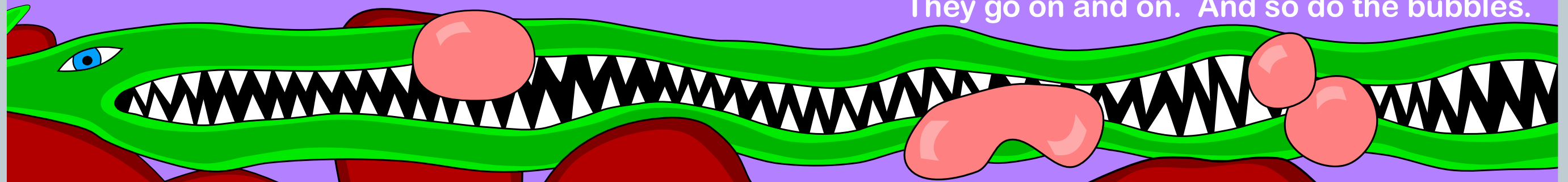
Hammerwood is an  
infinite crocodile. Let's  
zoom out to see him better.



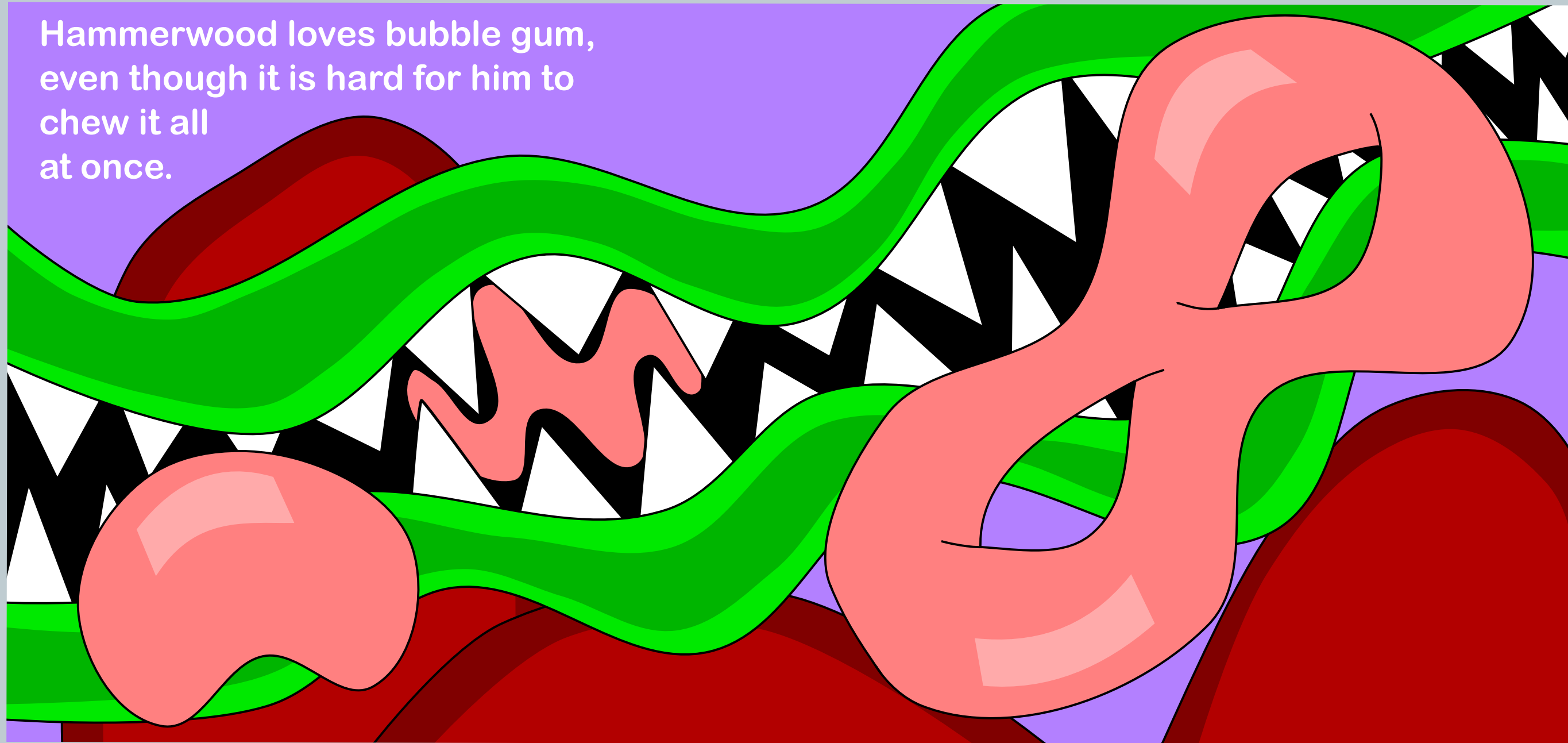
Look at those teeth!



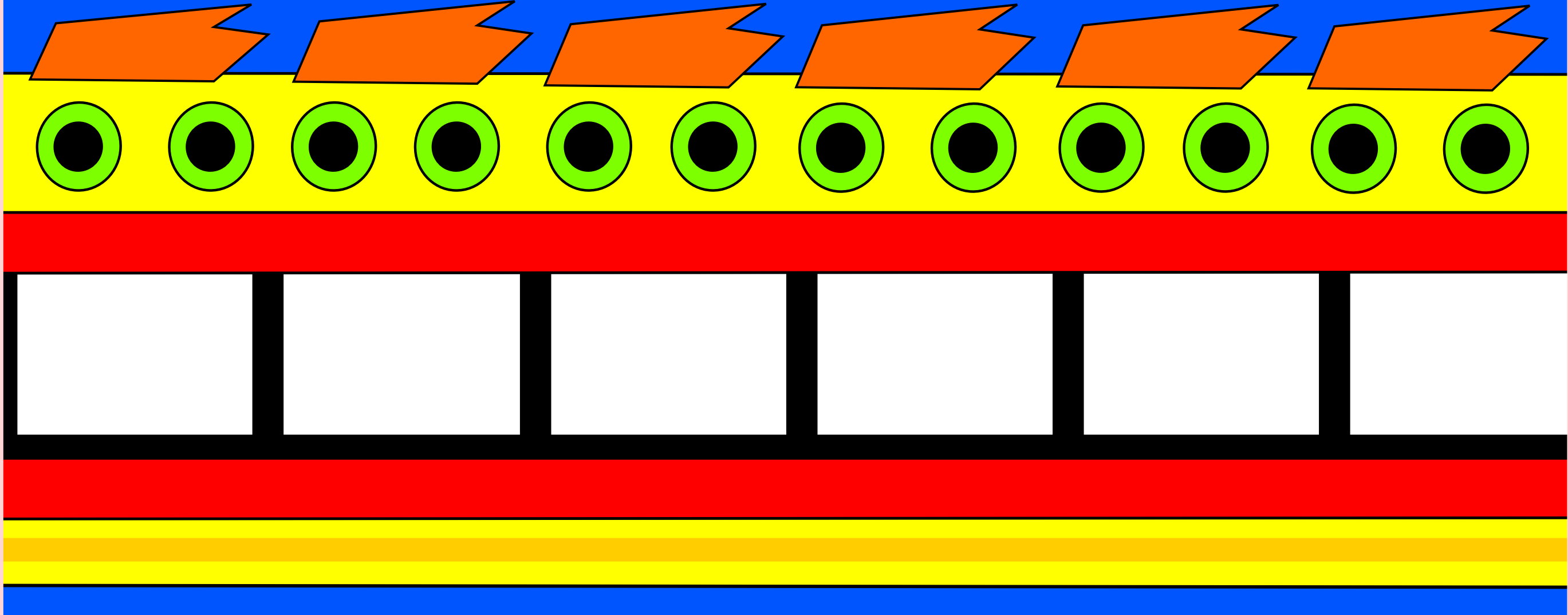
They go on and on. And so do the bubbles.

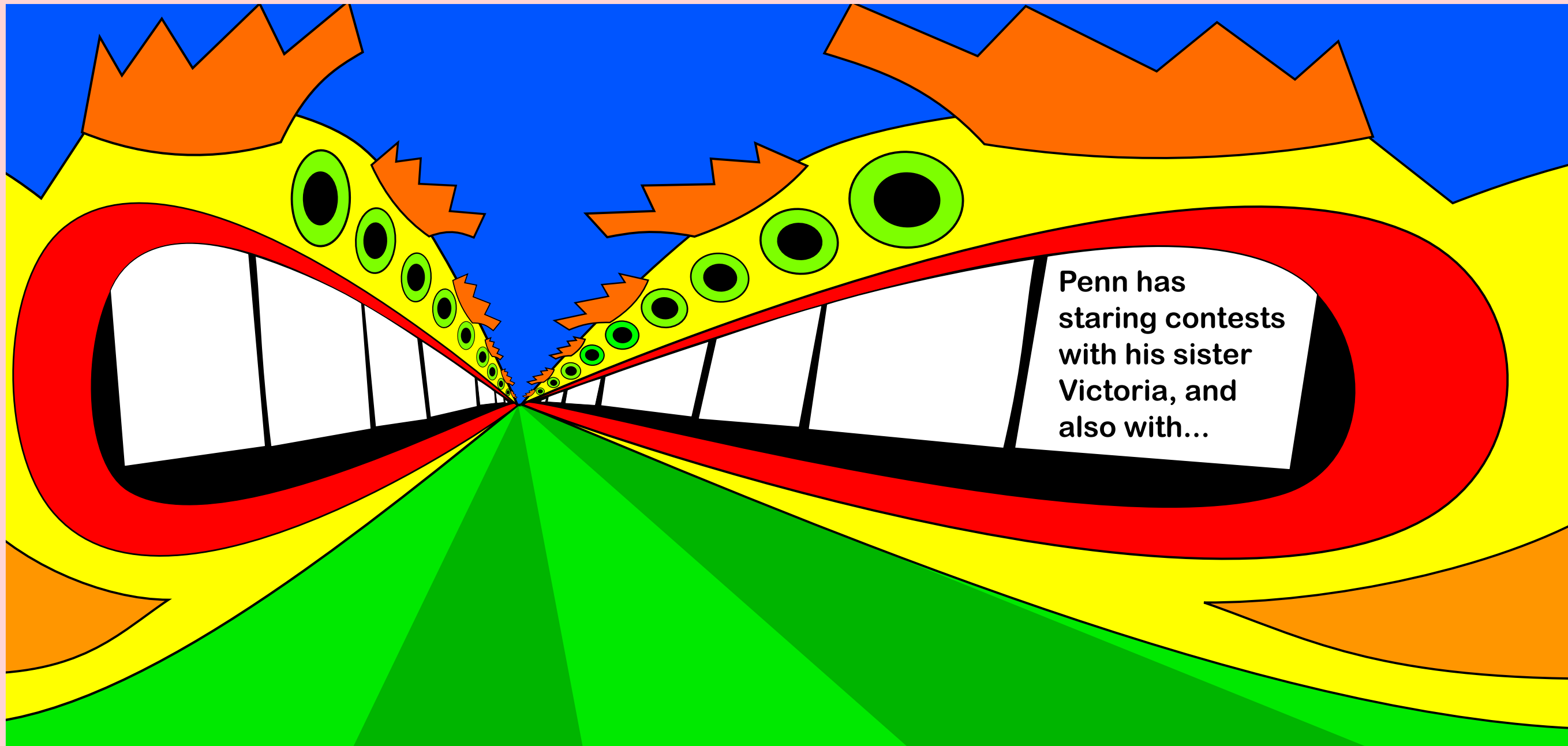


Hammerwood loves bubble gum,  
even though it is hard for him to  
chew it all  
at once.

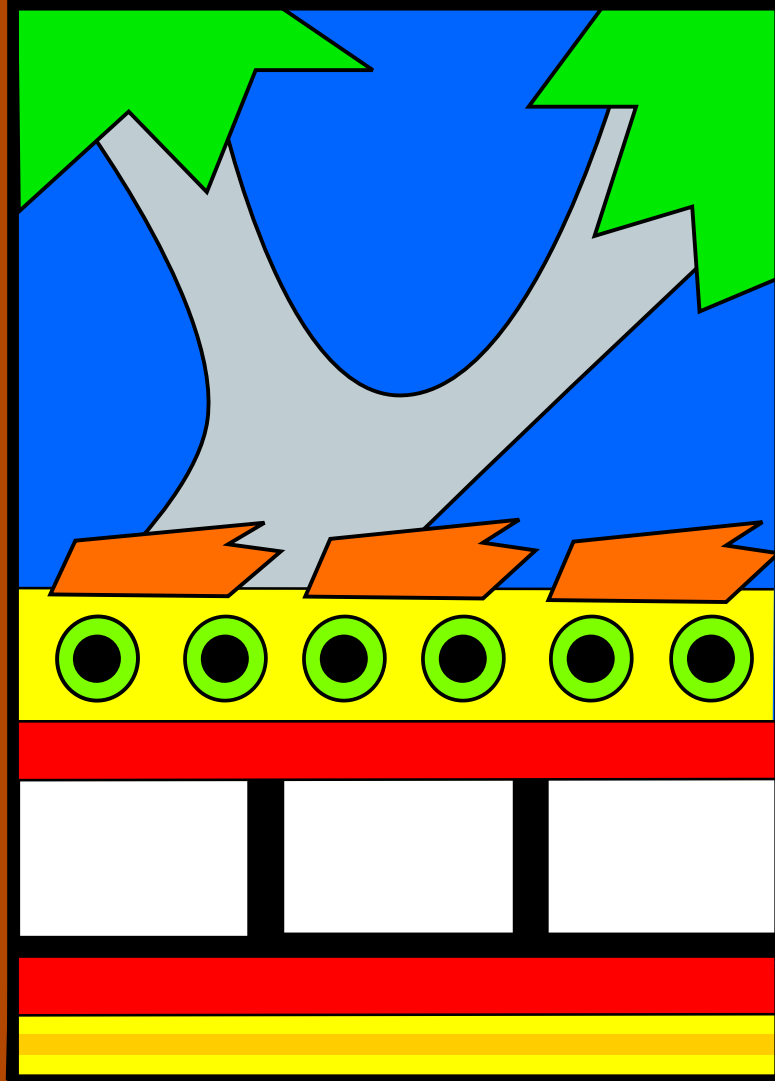


Penn also lives on the farm. He  
is an infinite chicken. On the infinite  
farm, chickens have teeth too.

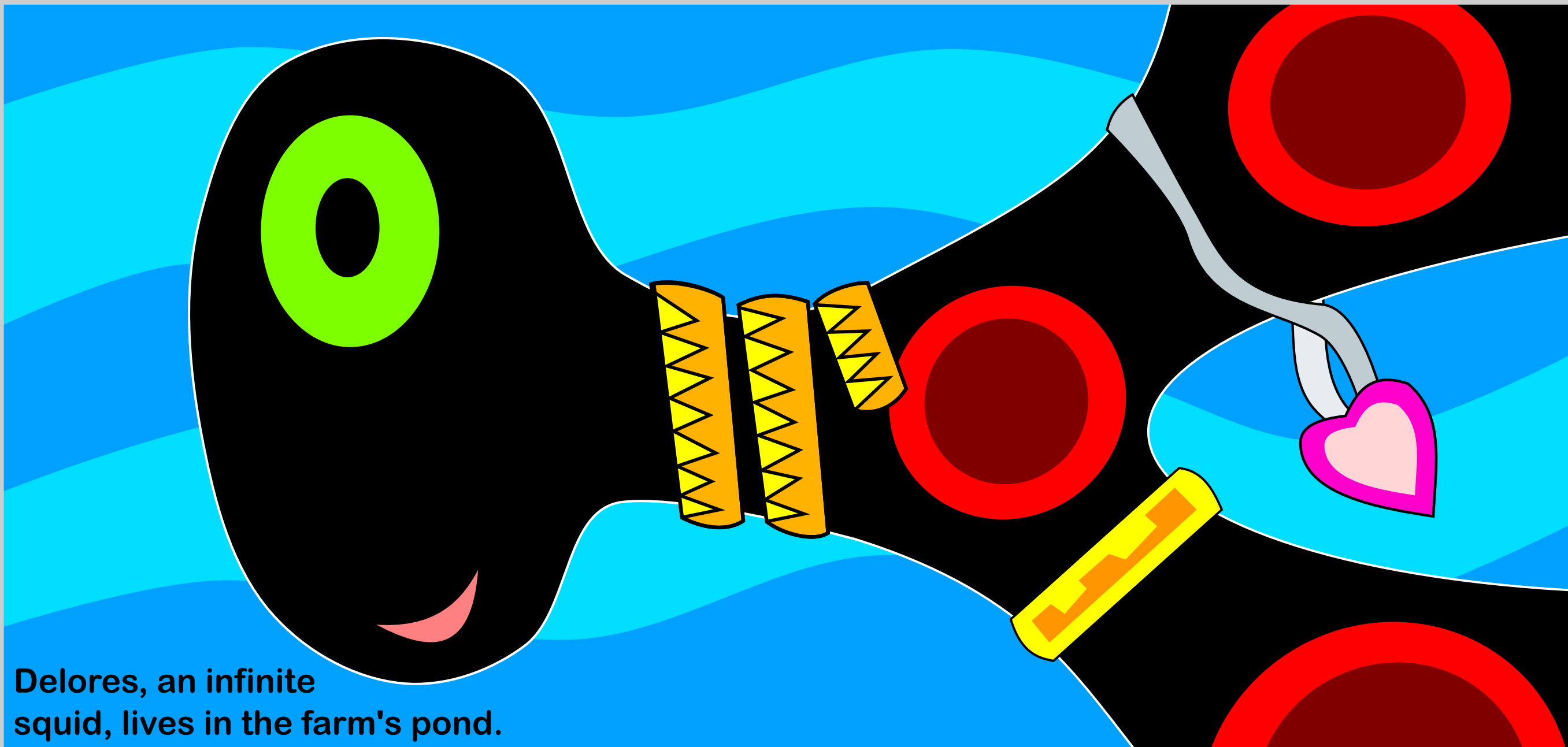




Bill, an owl with an  
infinite eye.

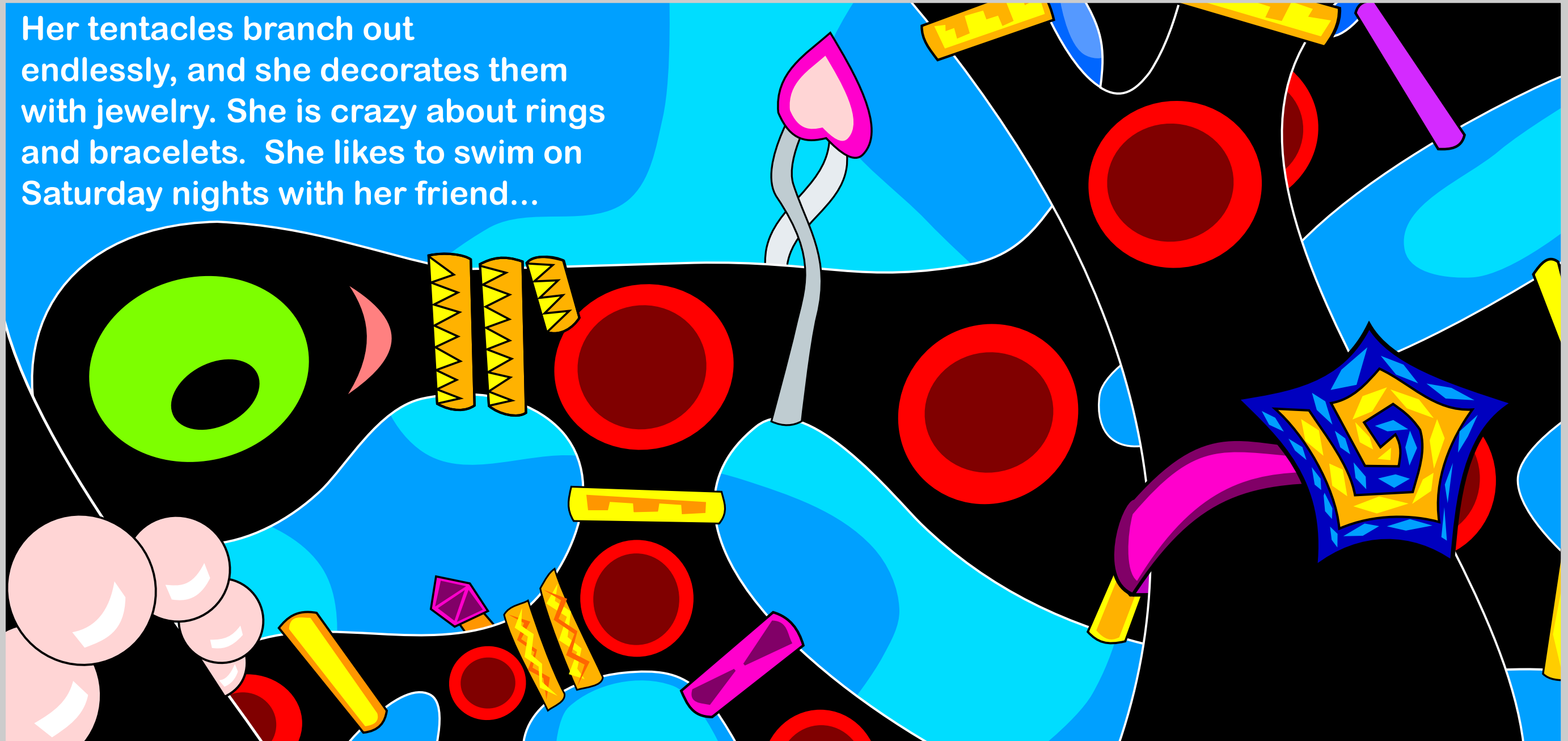


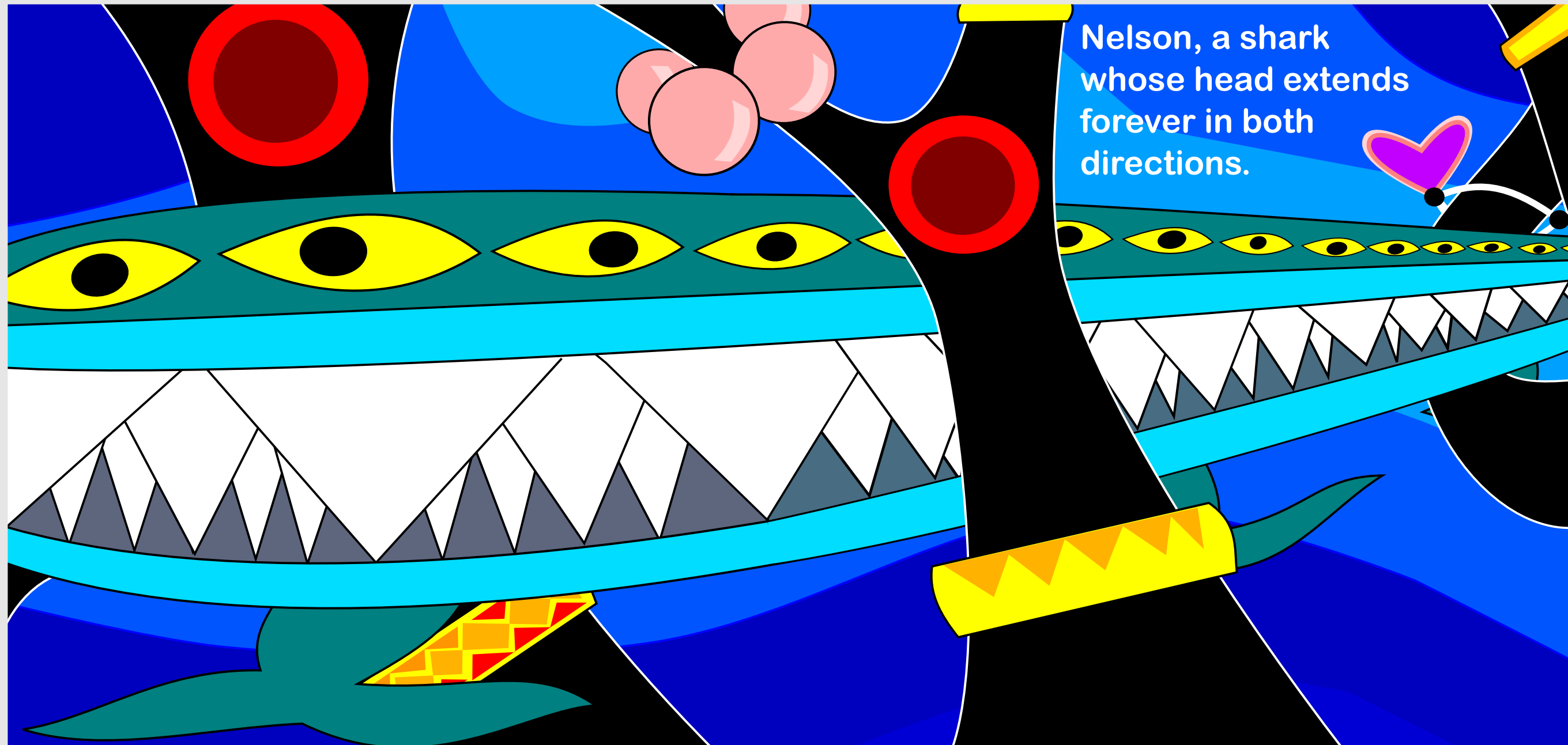




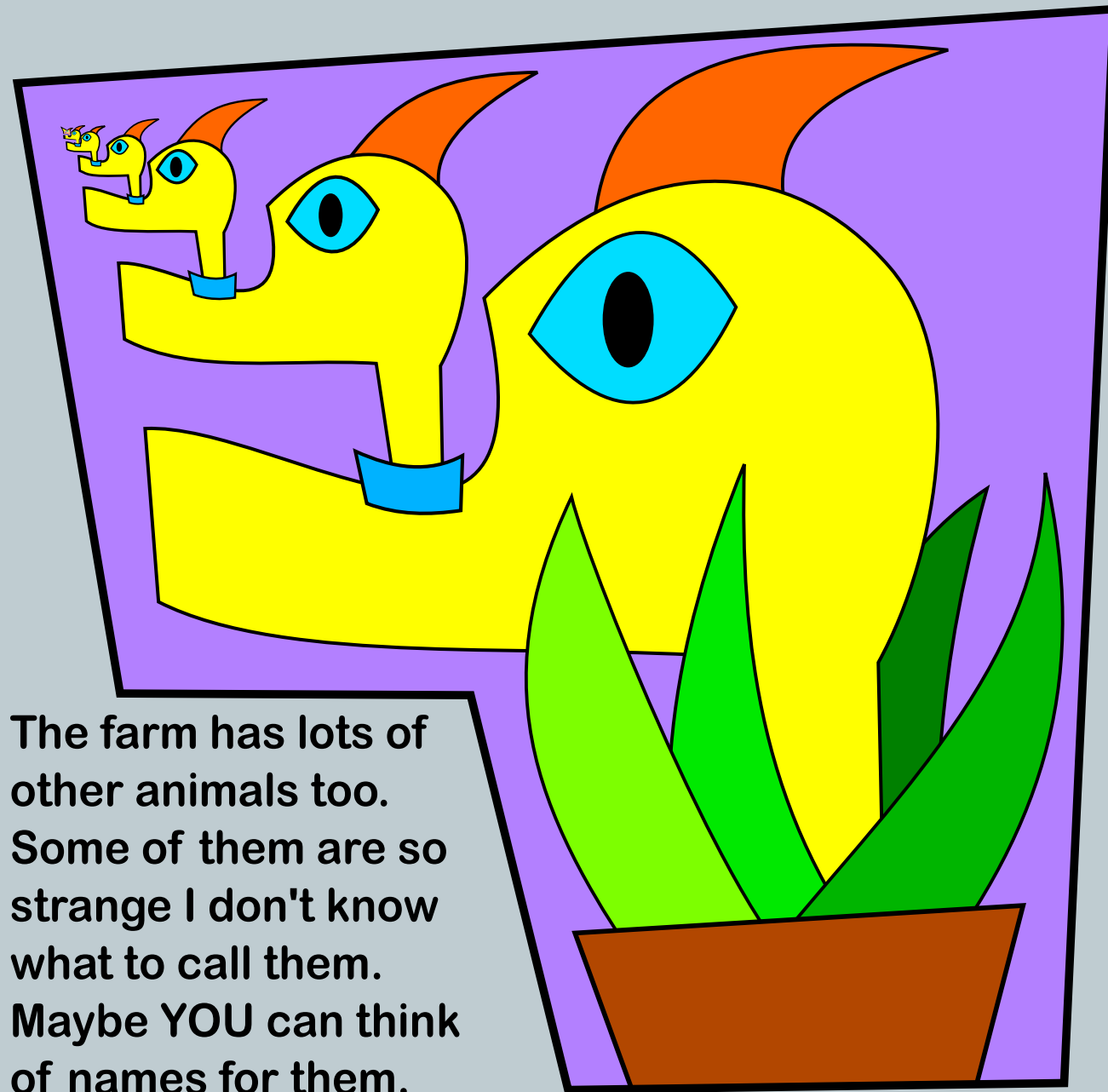
Delores, an infinite squid, lives in the farm's pond.

Her tentacles branch out endlessly, and she decorates them with jewelry. She is crazy about rings and bracelets. She likes to swim on Saturday nights with her friend...

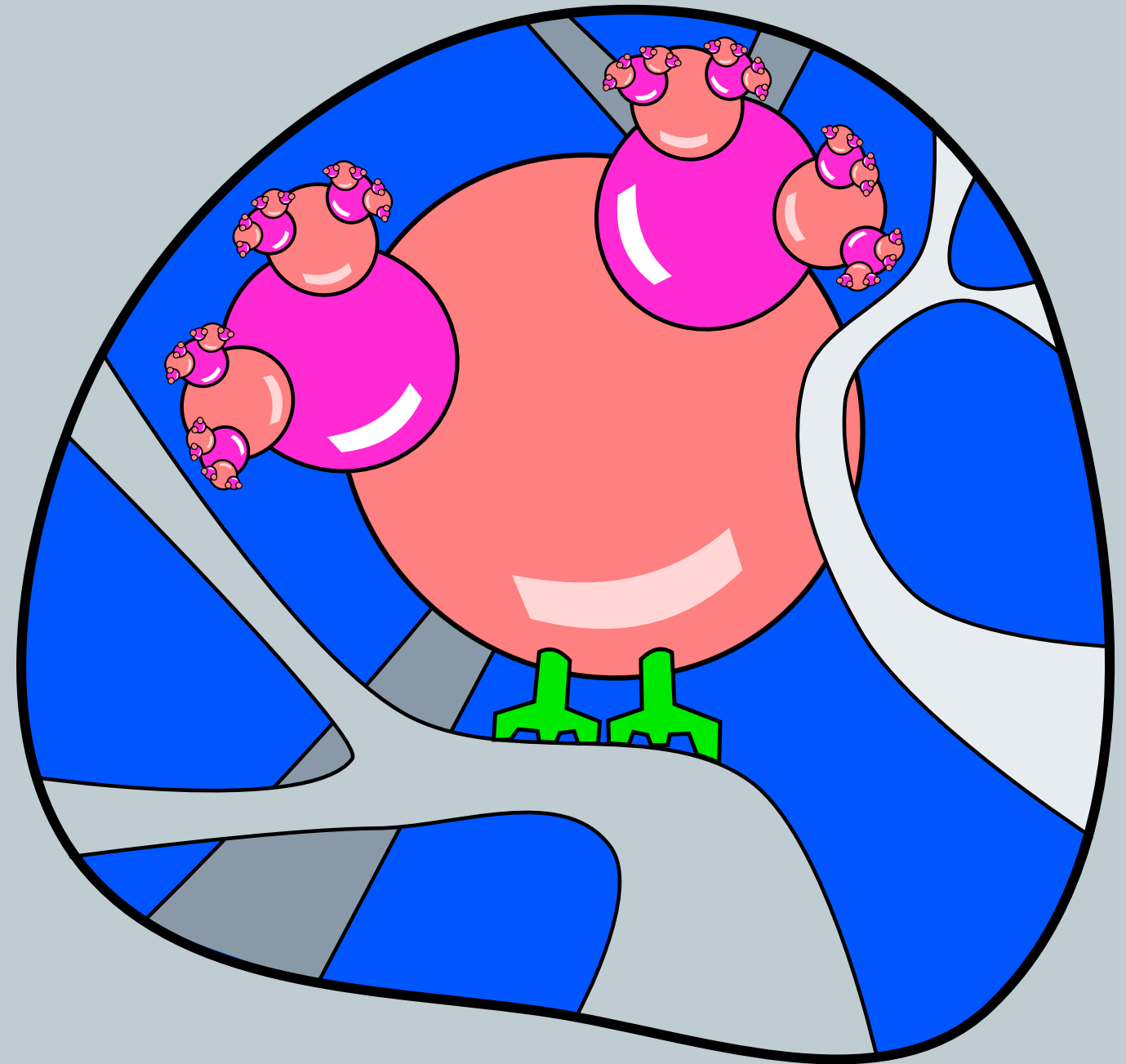




Nelson, a shark  
whose head extends  
forever in both  
directions.



The farm has lots of other animals too. Some of them are so strange I don't know what to call them. Maybe YOU can think of names for them.



**D**o the animals have any  
problems on the farm?

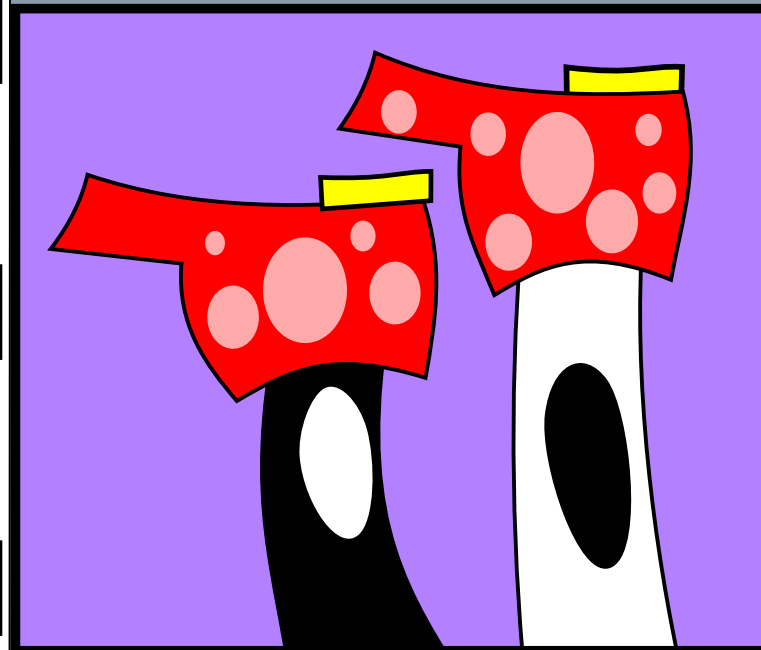
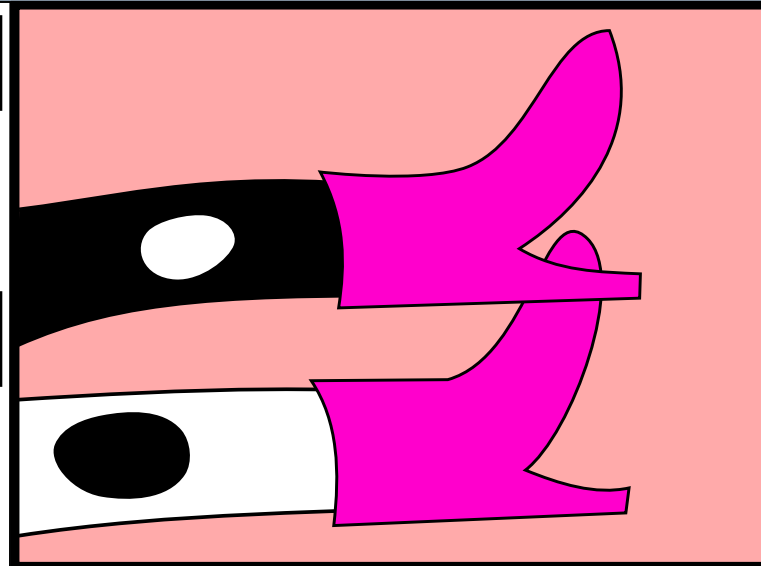
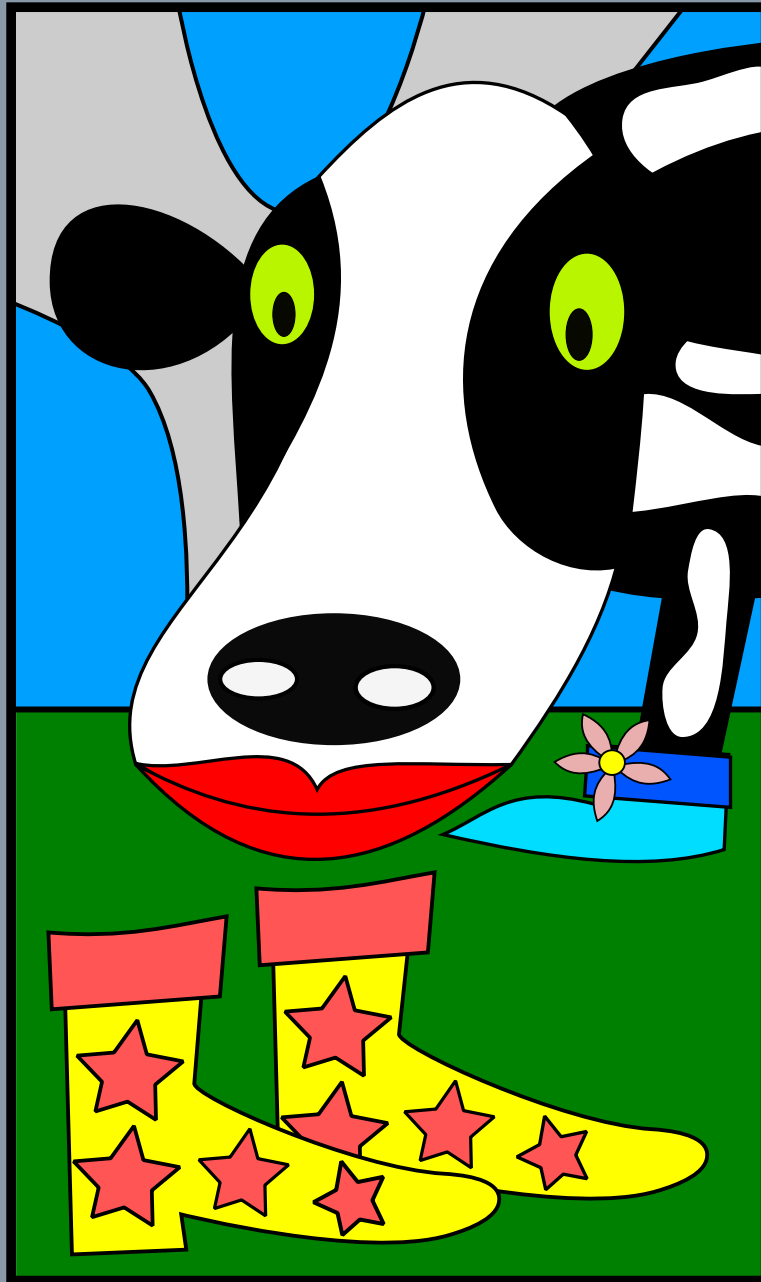




Oh yes,  
lots of  
problems!

Gracie has  
a problem  
with shoes.  
Her friends  
know that  
she loves  
shoes, so  
they bring  
her new  
ones as  
presents.







Hammerwood has  
problems with  
his teeth.

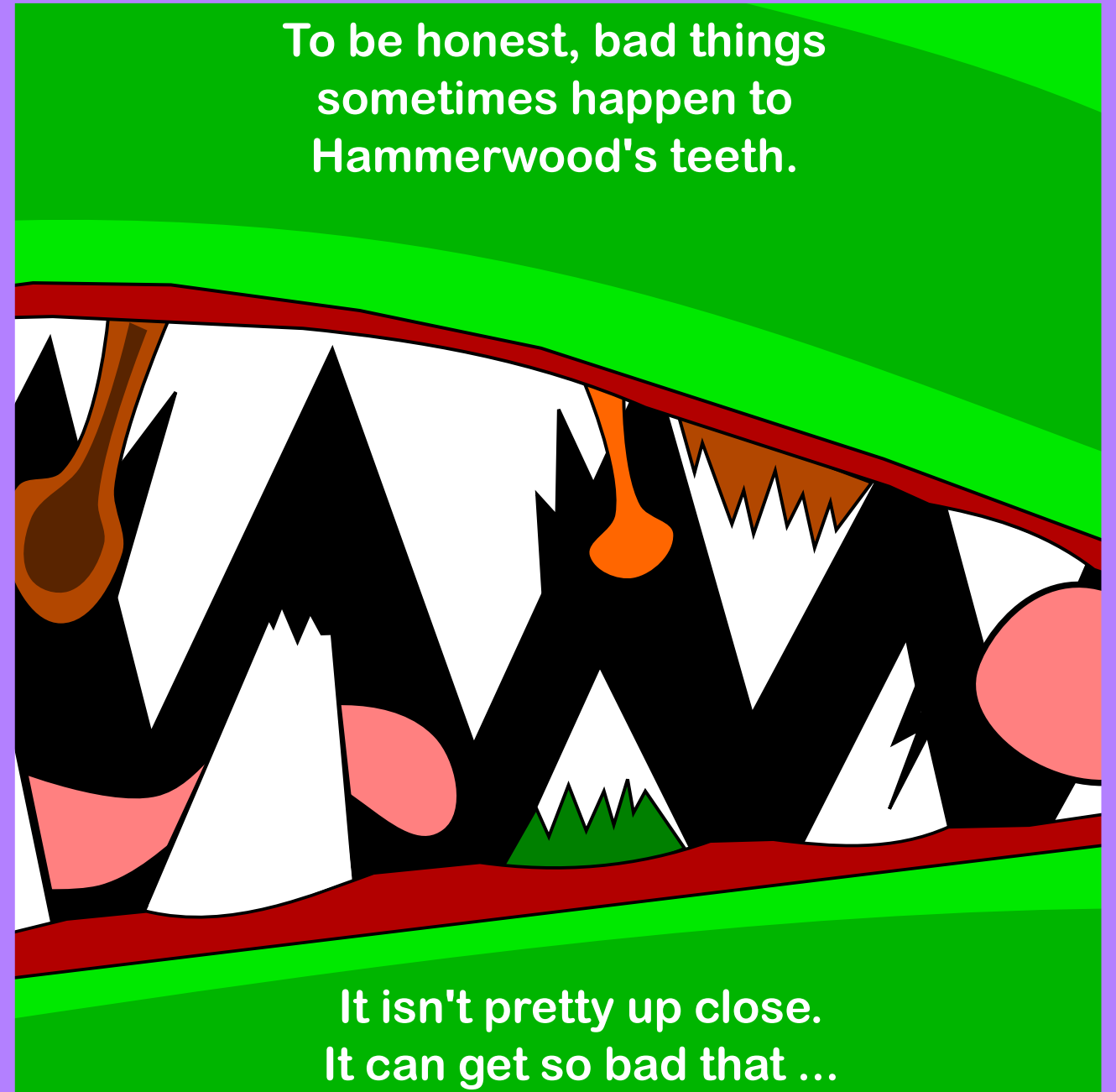


He has so much fun  
blowing bubbles ...

...that he often forgets  
to brush his teeth.



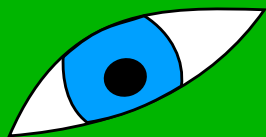
To be honest, bad things  
sometimes happen to  
Hammerwood's teeth.



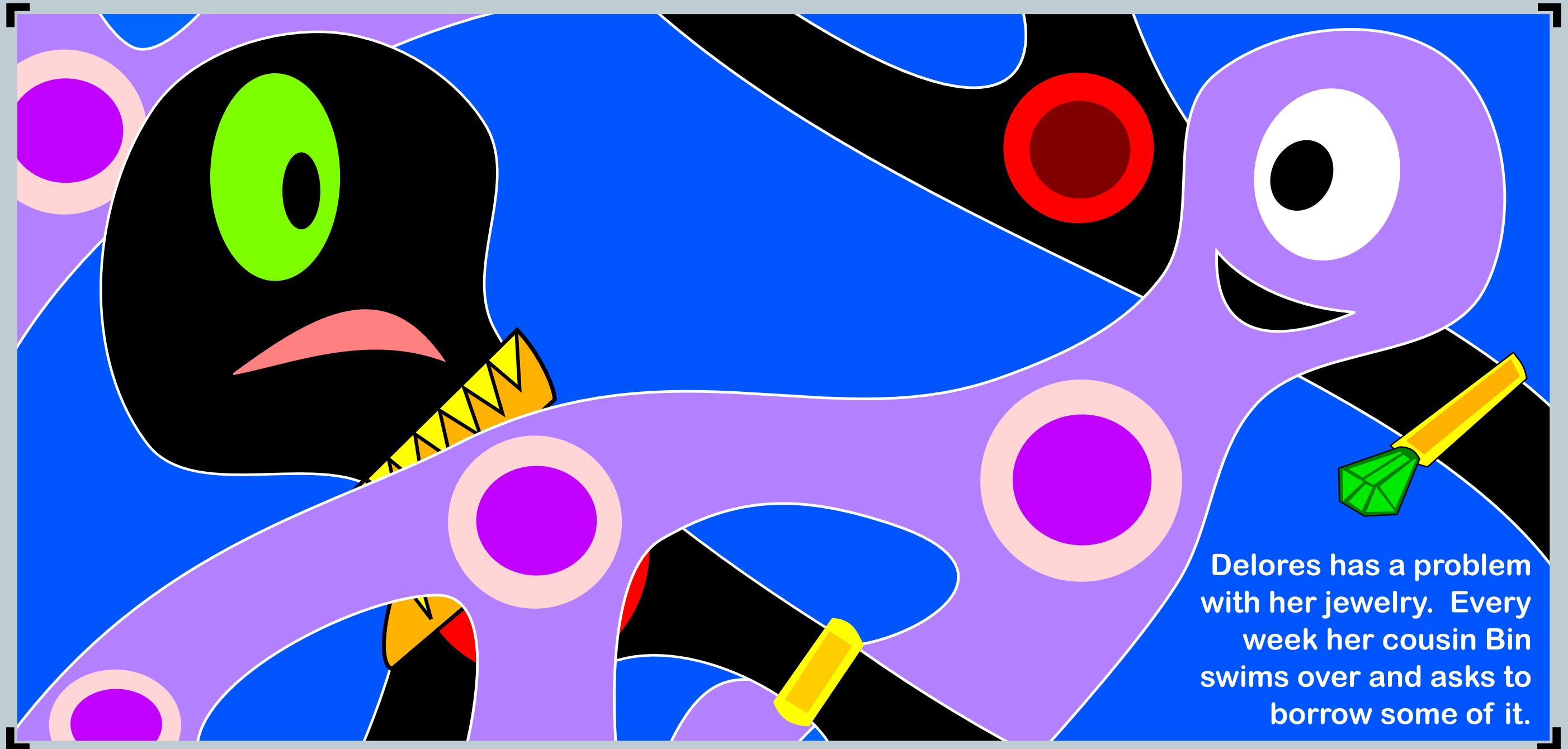
It isn't pretty up close.  
It can get so bad that ...

he loses most of *his* teeth.

He needs those teeth for chewing  
and he can't grow new ones.



How can he fix his teeth?



Delores has a problem  
with her jewelry. Every  
week her cousin Bin  
swims over and asks to  
borrow some of it.

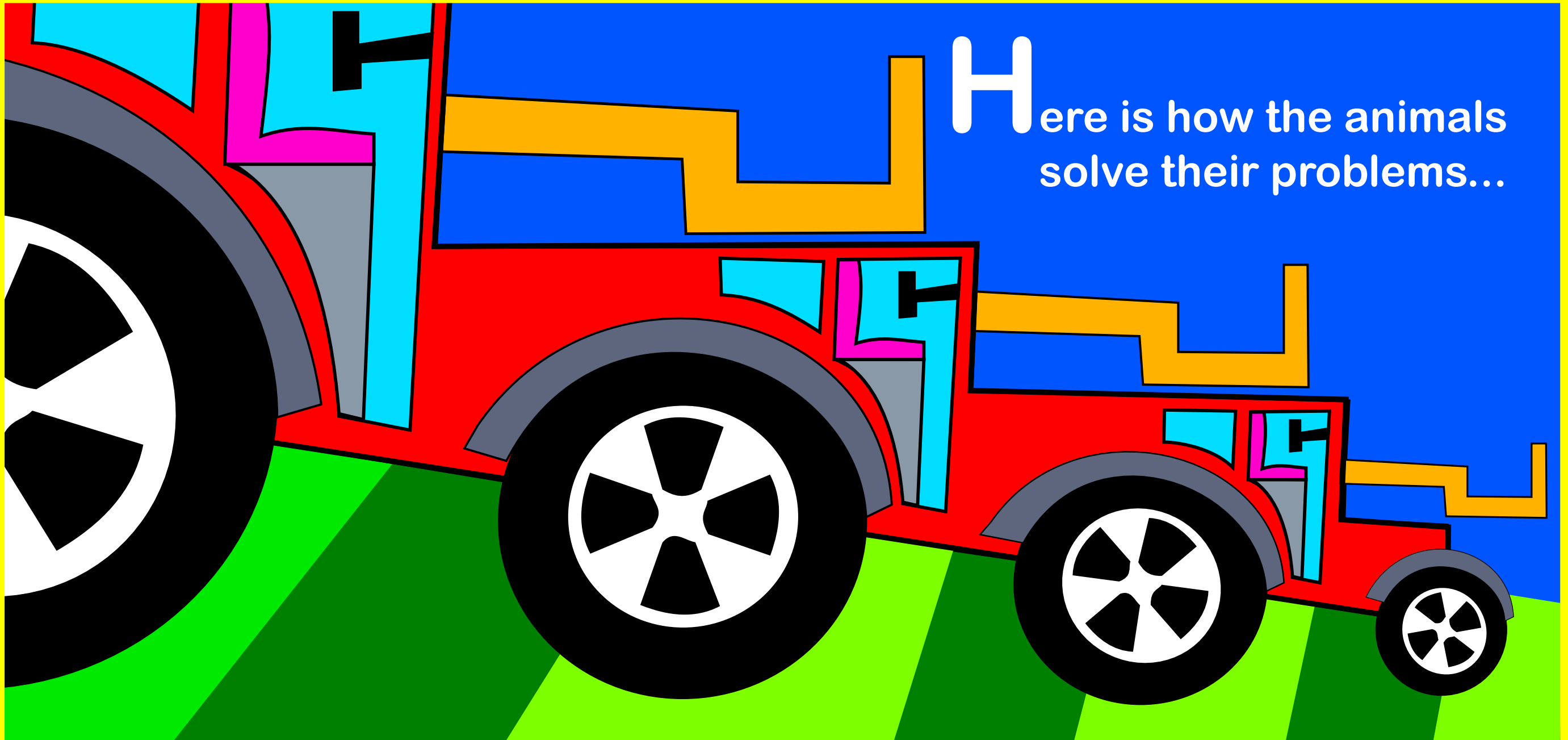


Bin wants to  
decorate all his  
tentacles for his  
Saturday night dances...

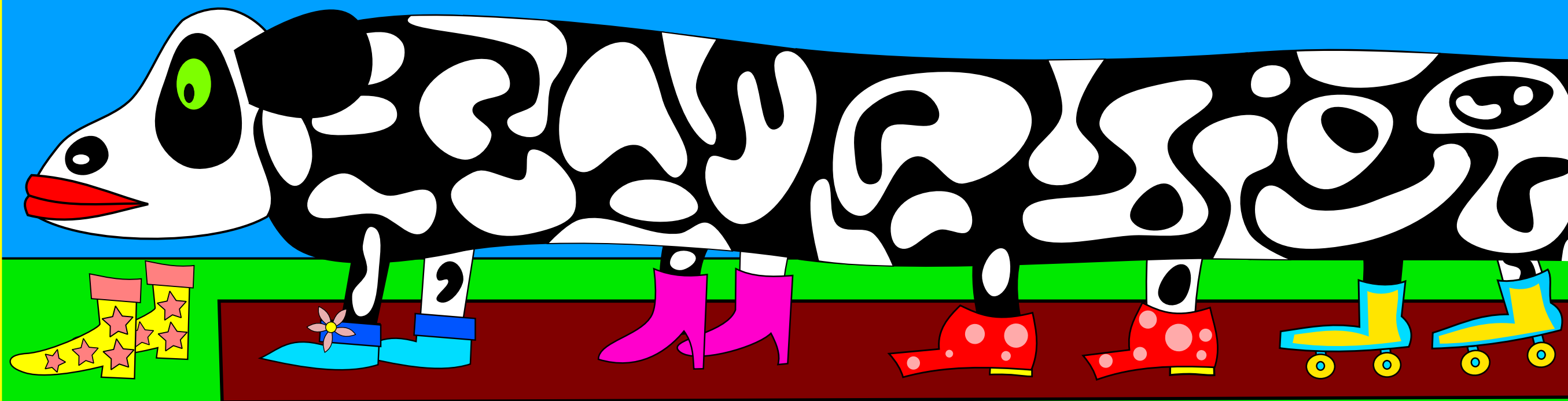


but Delores wants to decorate all of HER tentacles for her Saturday night swims with Nelson. How can they split the jewelry so that each has enough for all of their tentacles?

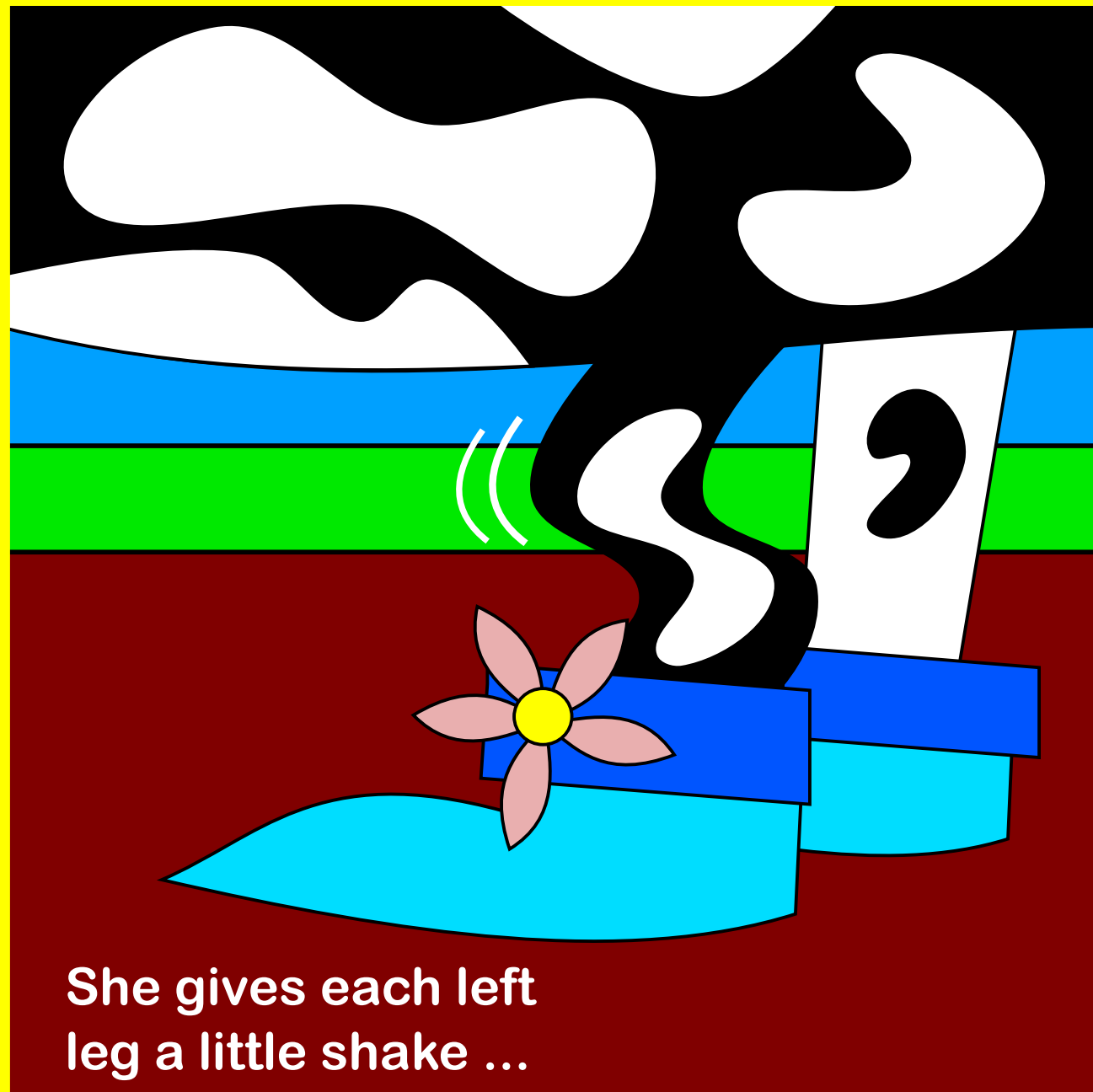
Here is how the animals  
solve their problems...



Gracie prepares to put on her new shoes by standing behind them on an infinitely long sticky strip.







She gives each left  
leg a little shake ...

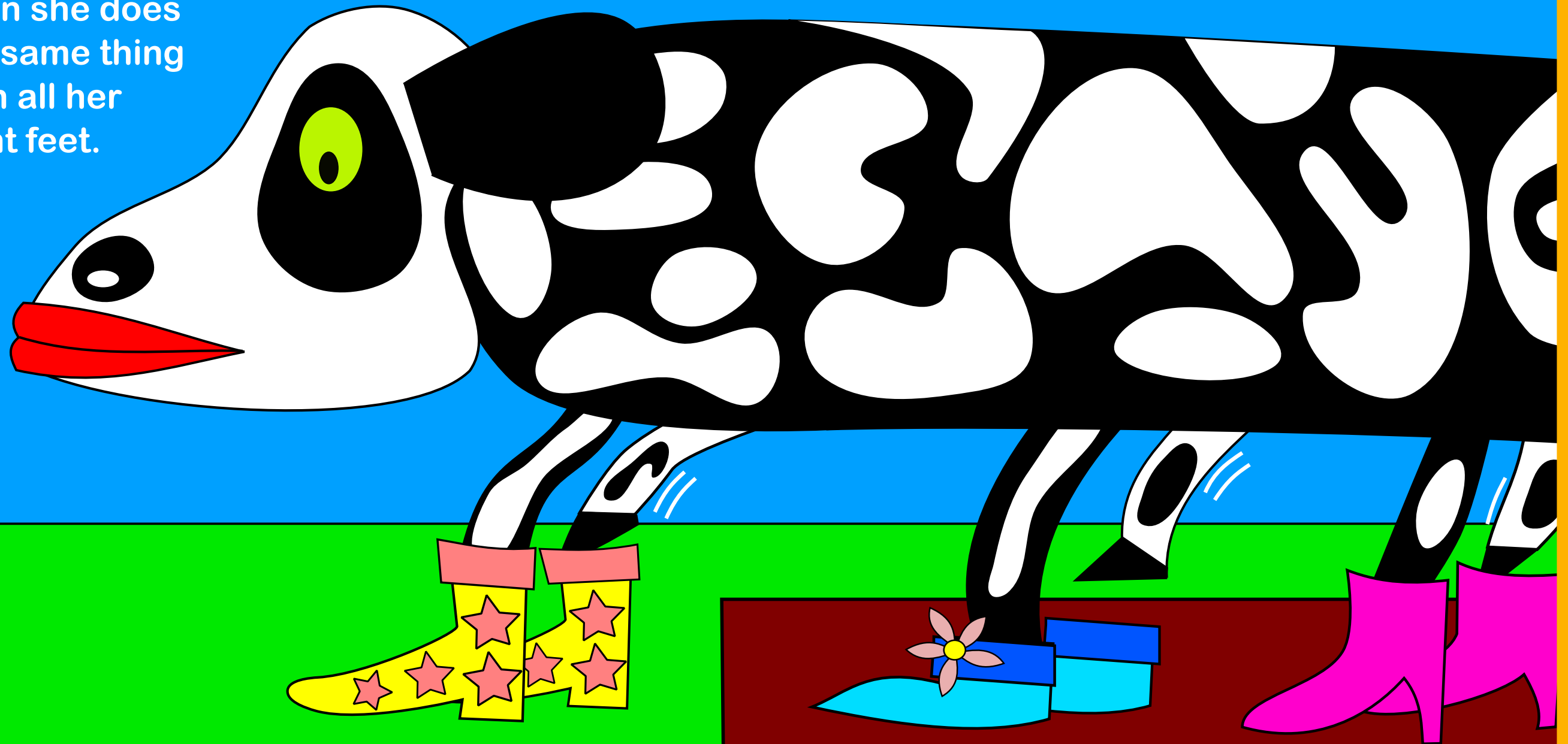


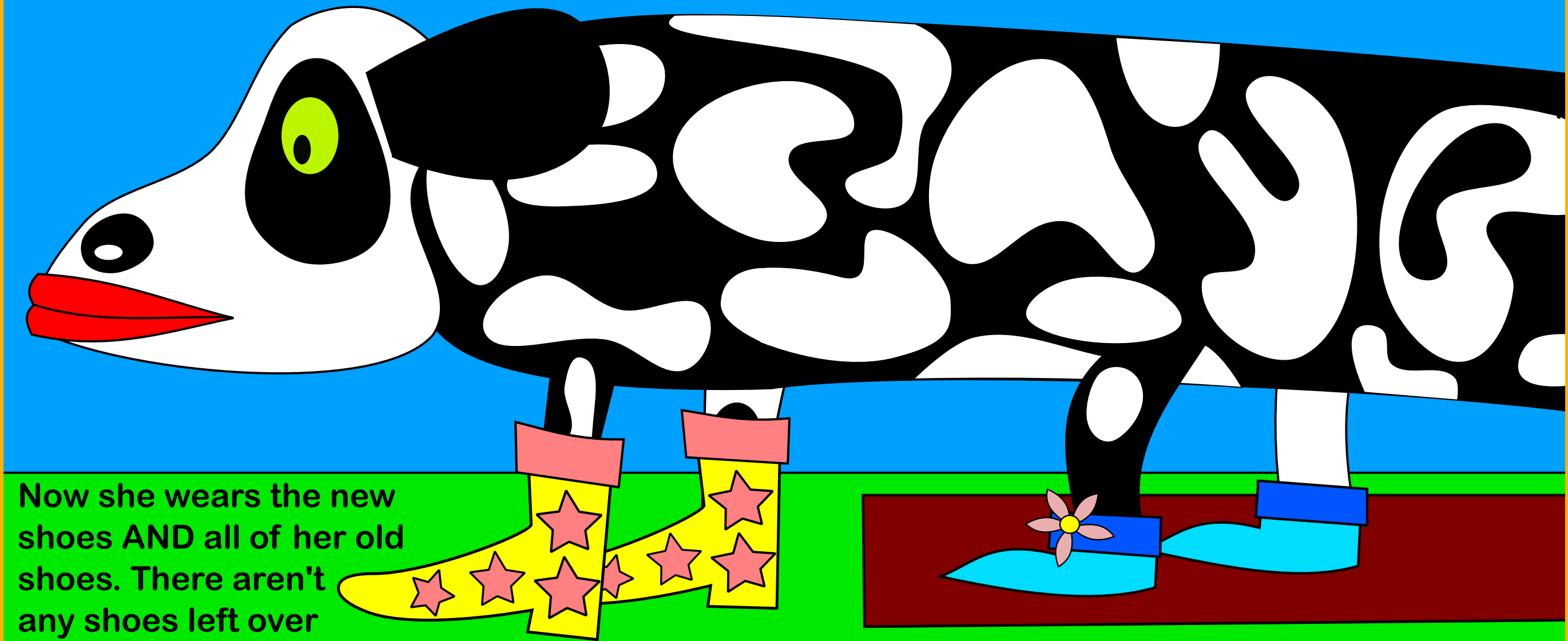
then steps out of  
each left shoe ...

and into the one  
in front of it.



Then she does  
the same thing  
with all her  
right feet.

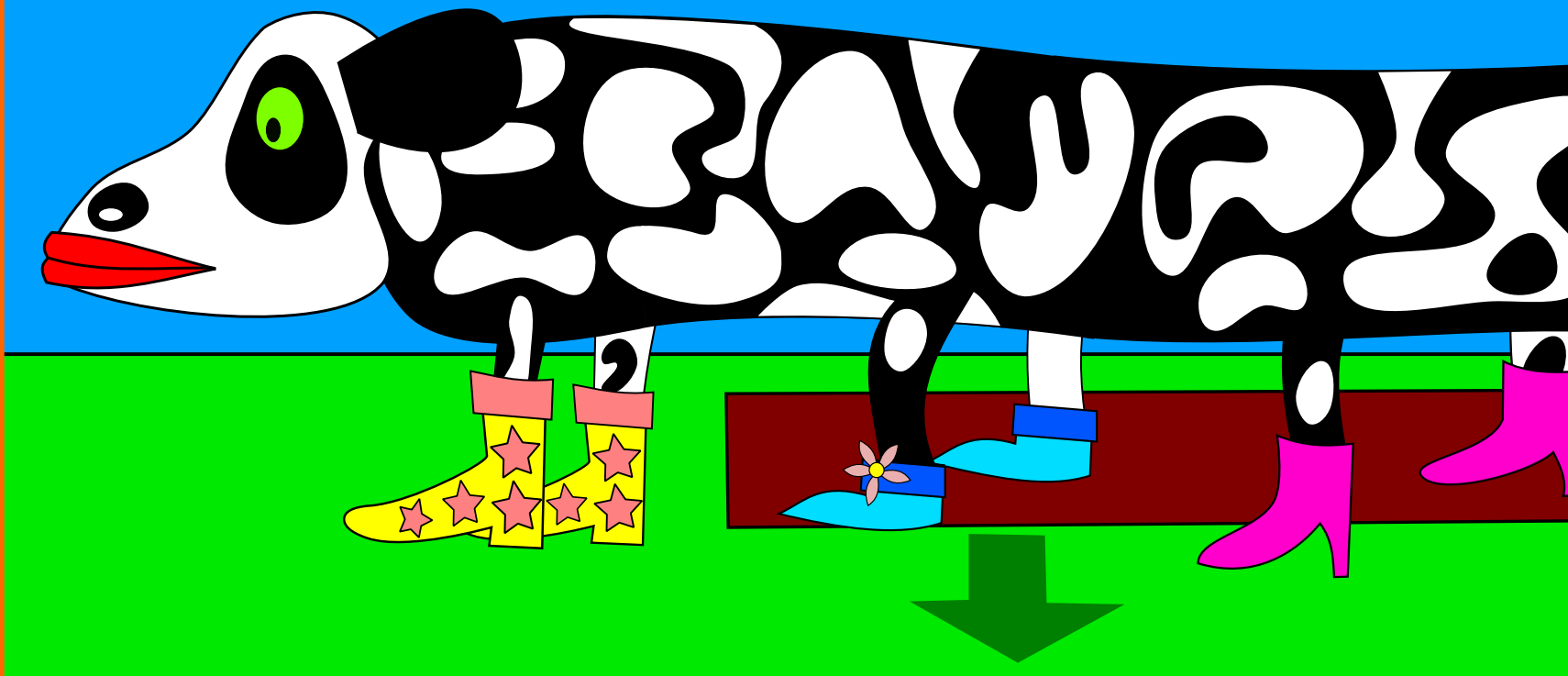




Now she wears the new shoes AND all of her old shoes. There aren't any shoes left over at the back end because...

Gracie doesn't have a back end!

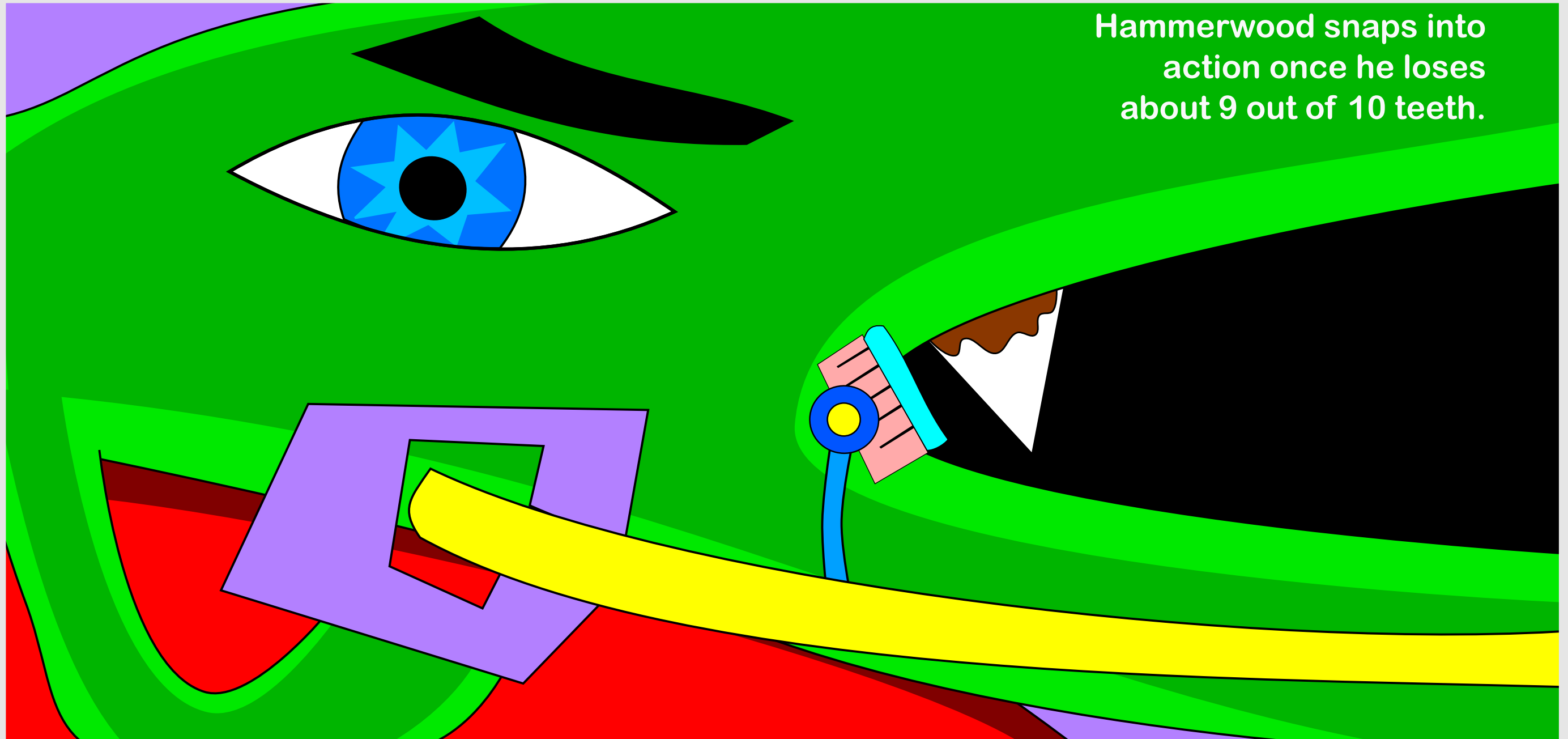
Gracie moves off the  
sticky tape and then...



she is ready for another present.

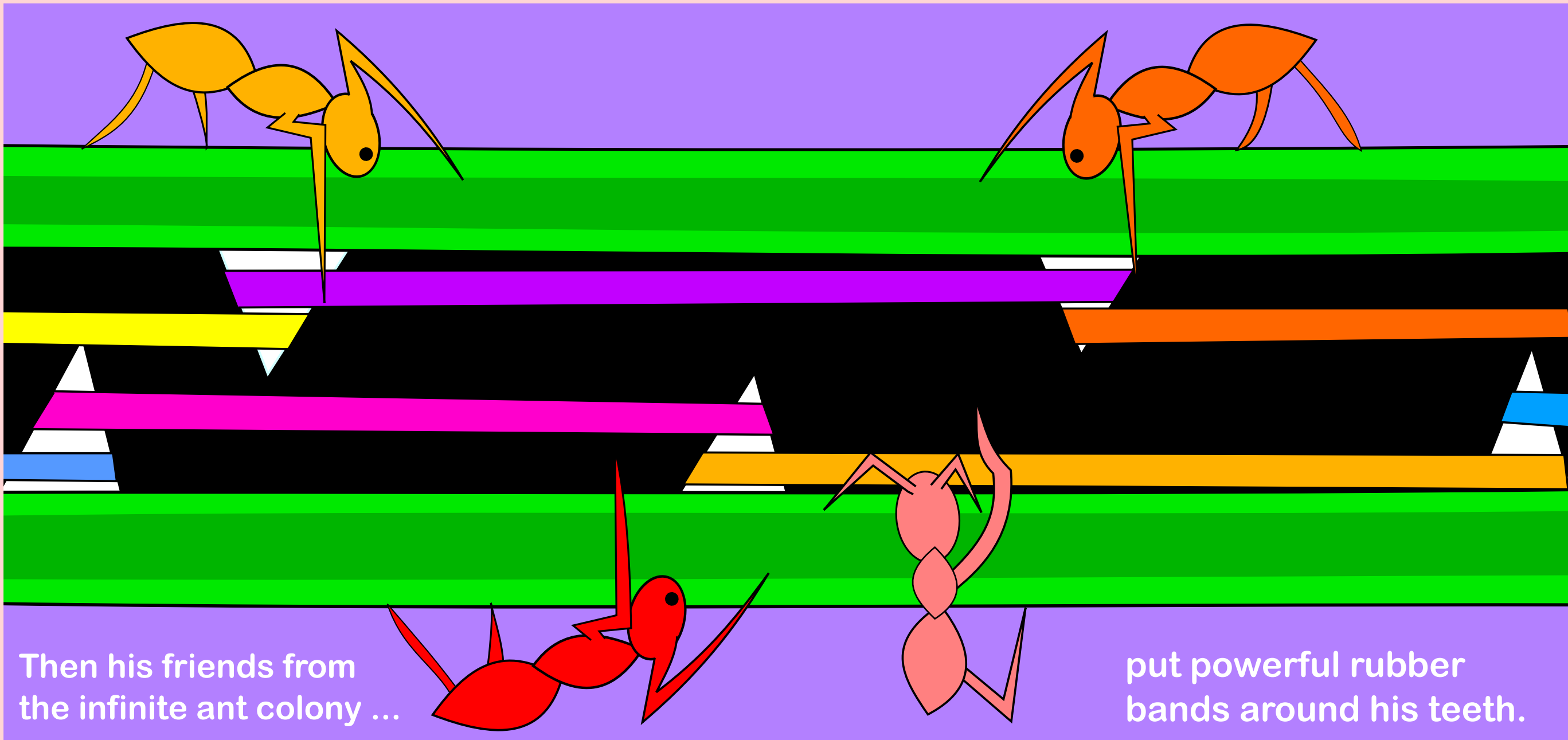


Hammerwood snaps into  
action once he loses  
about 9 out of 10 teeth.



The first thing he  
does is brush his  
remaining teeth.

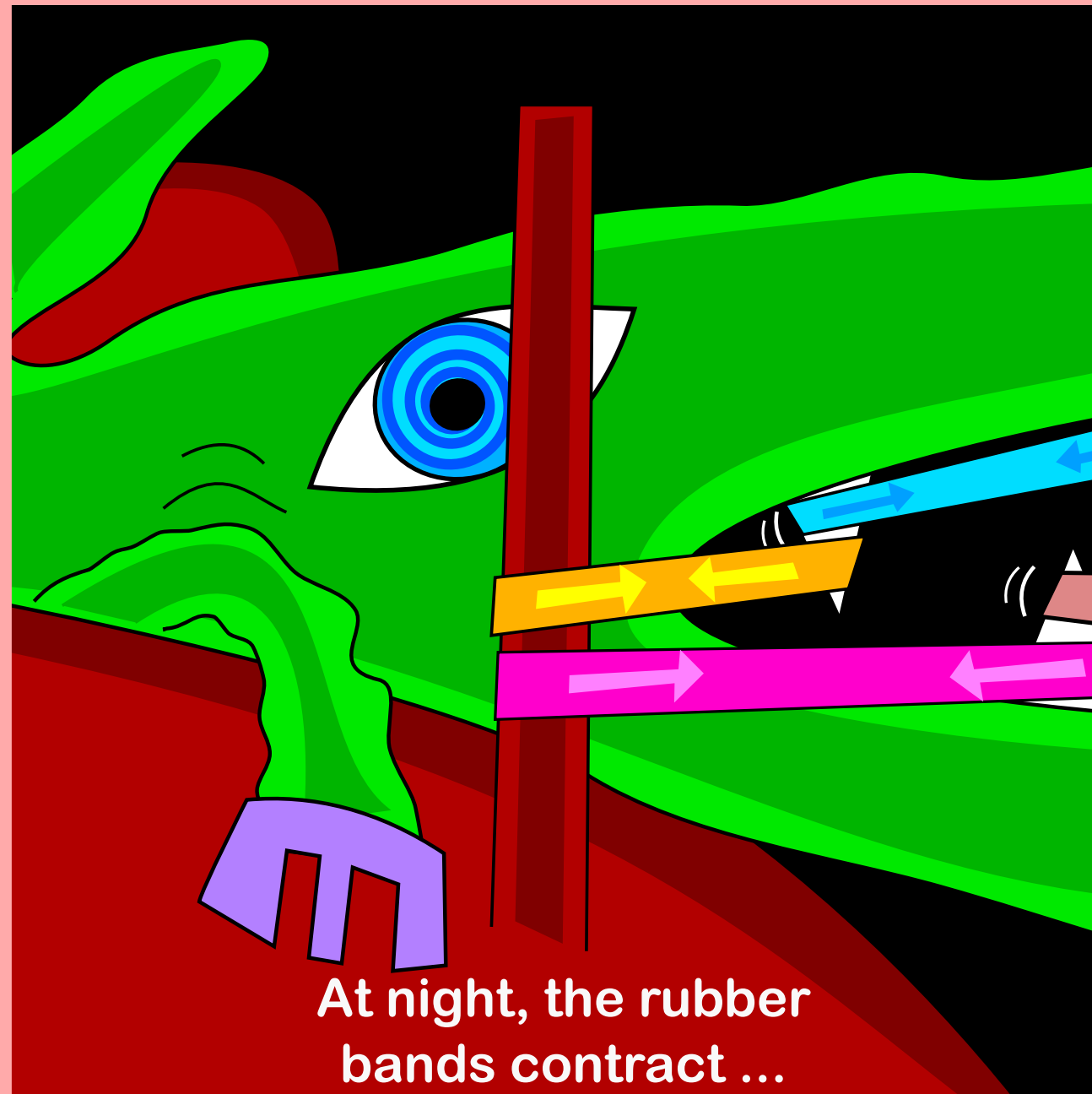
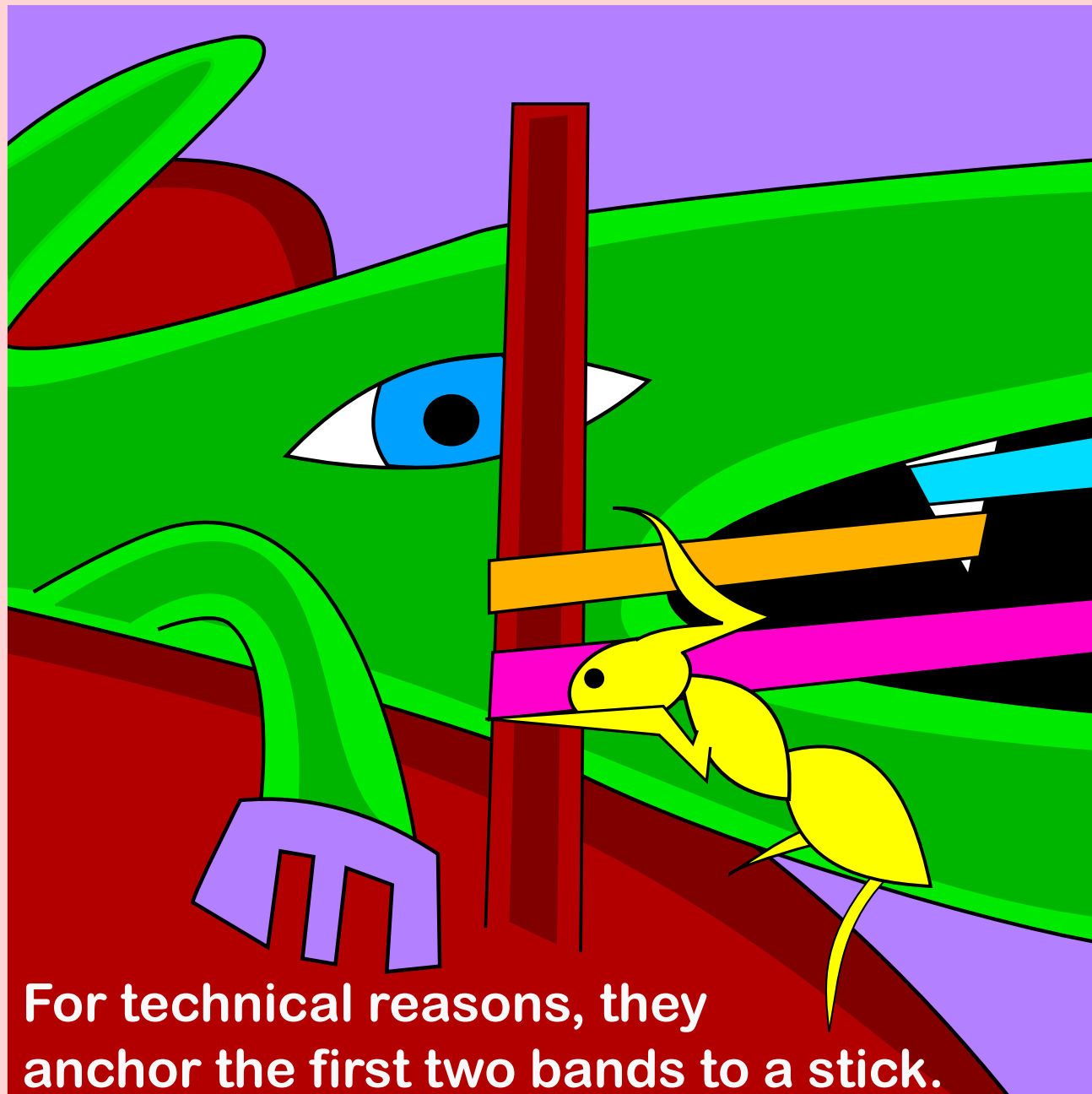


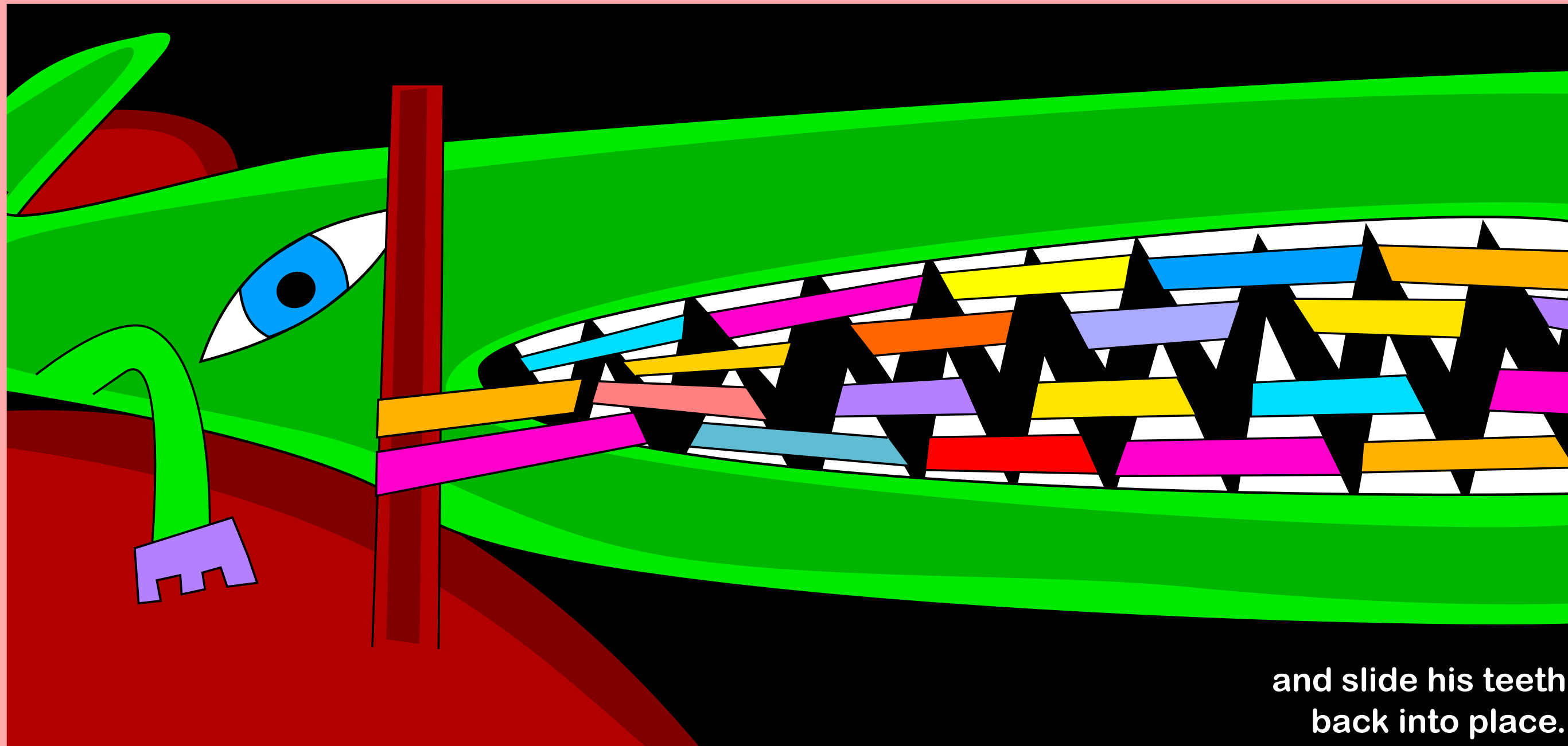


Then his friends from  
the infinite ant colony ...

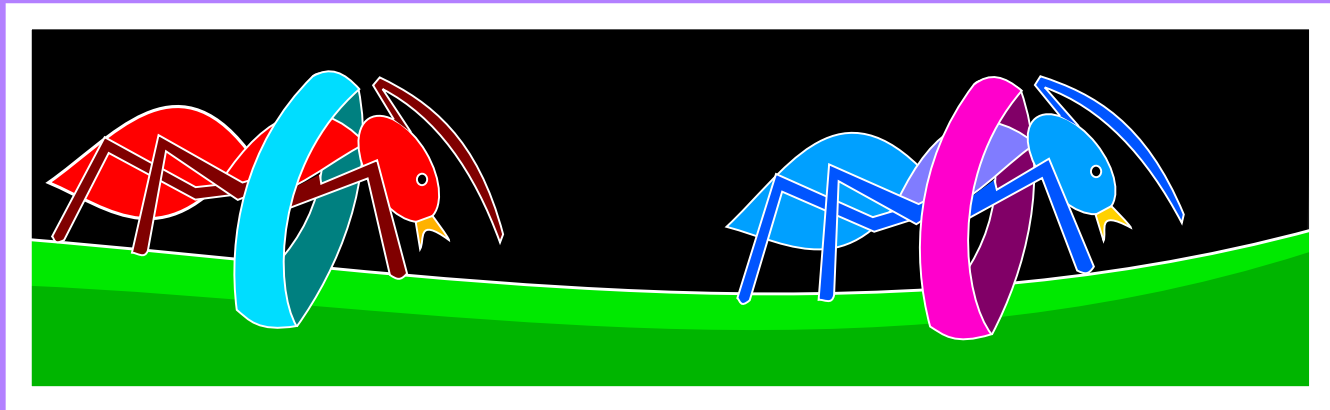
put powerful rubber  
bands around his teeth.



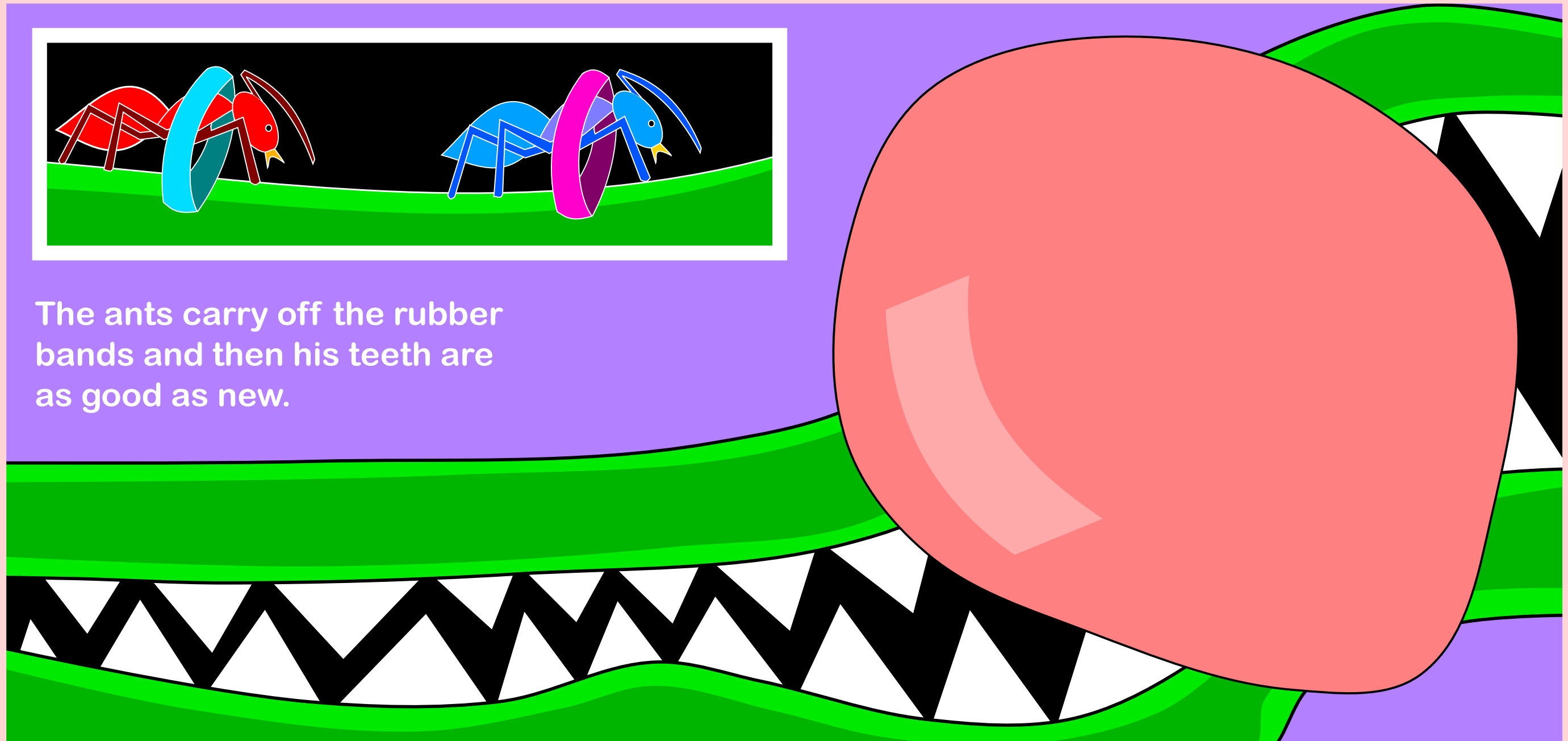




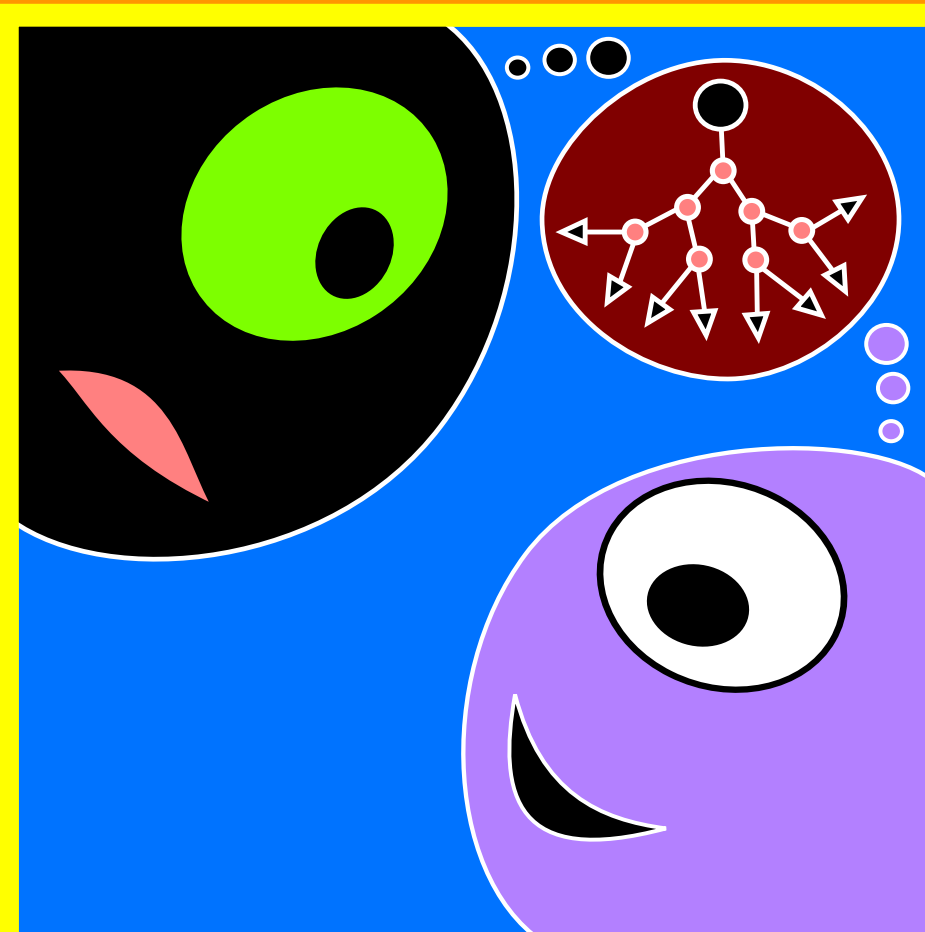
and slide his teeth  
back into place.



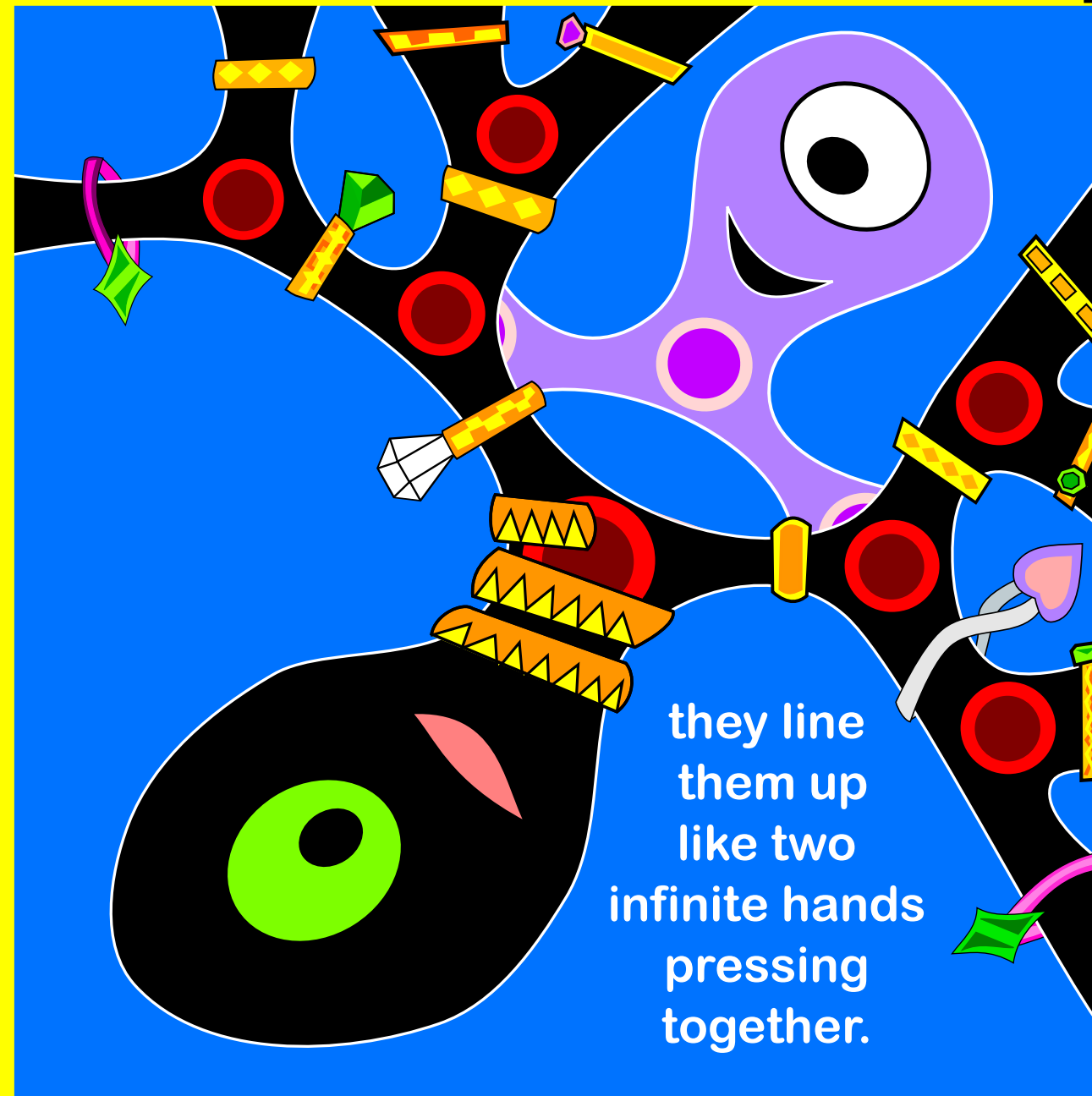
The ants carry off the rubber bands and then his teeth are as good as new.



Delores agrees to lend  
Bin some jewelry.



Their tentacles branch out in the  
same way, so on Saturday afternoon...

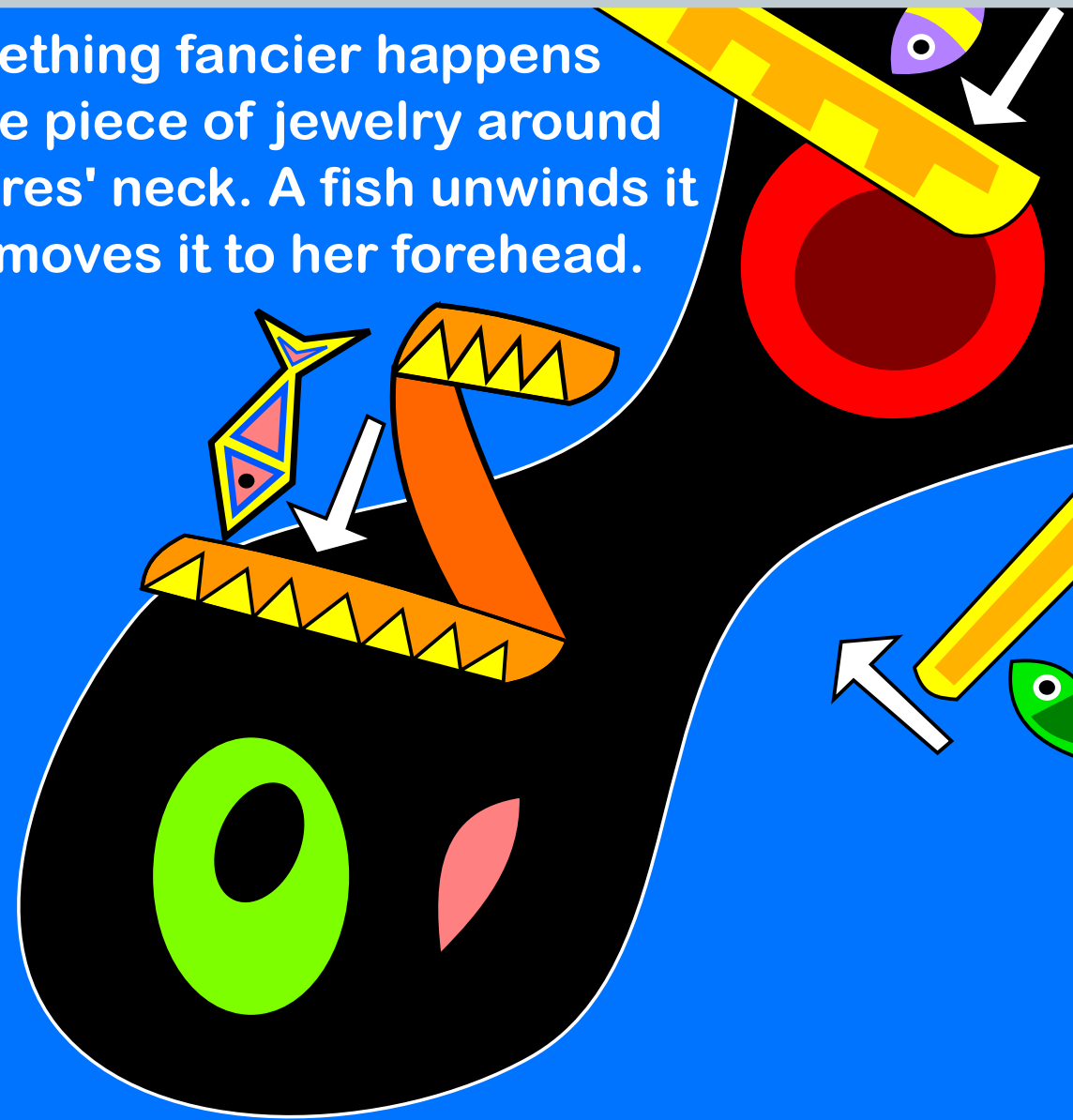




[illegible]

and is replaced by TWO pieces that come from behind it!

Something fancier happens  
to the piece of jewelry around  
Delores' neck. A fish unwinds it  
and moves it to her forehead.



It becomes her  
tiara for  
the evening.

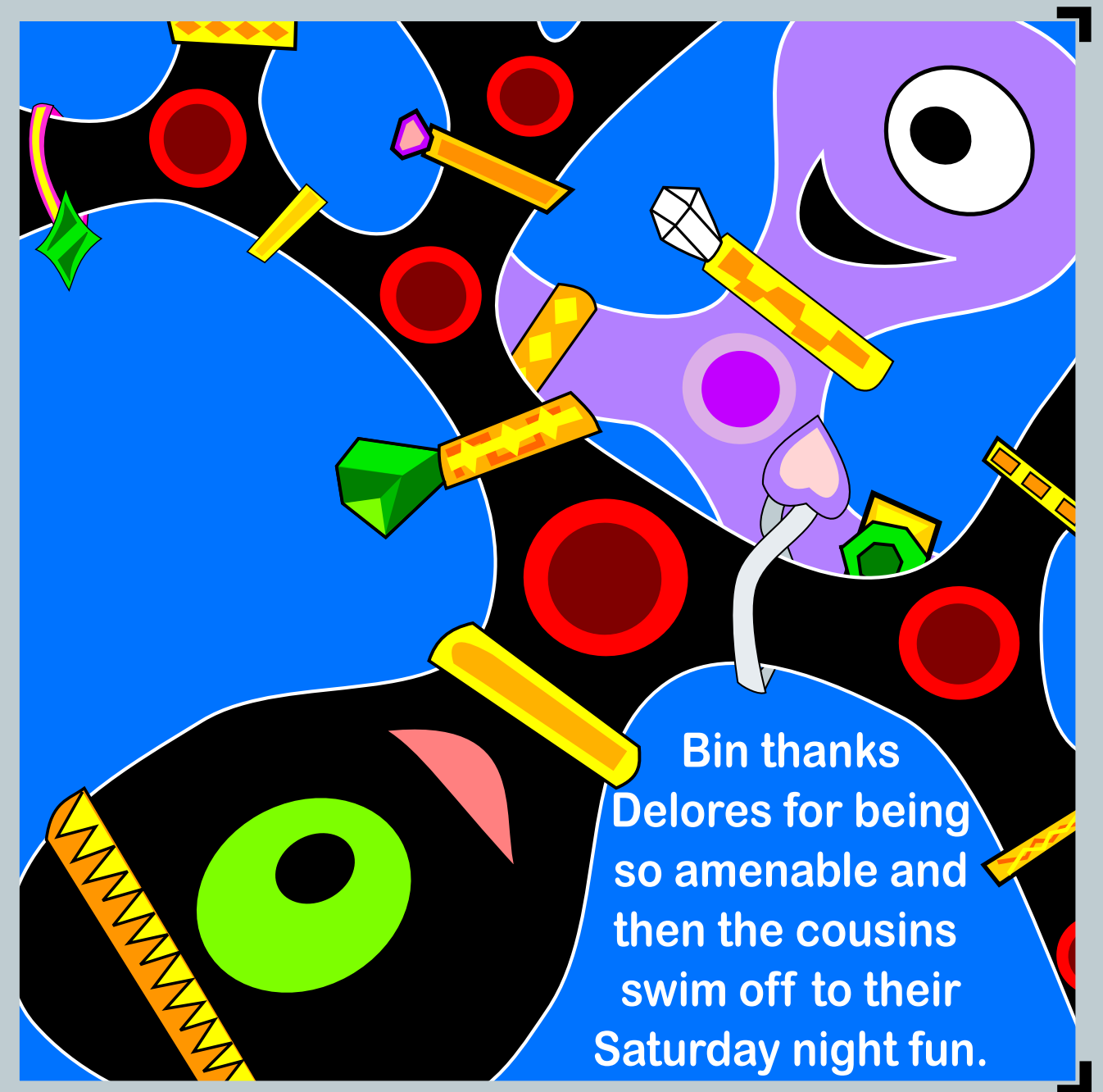
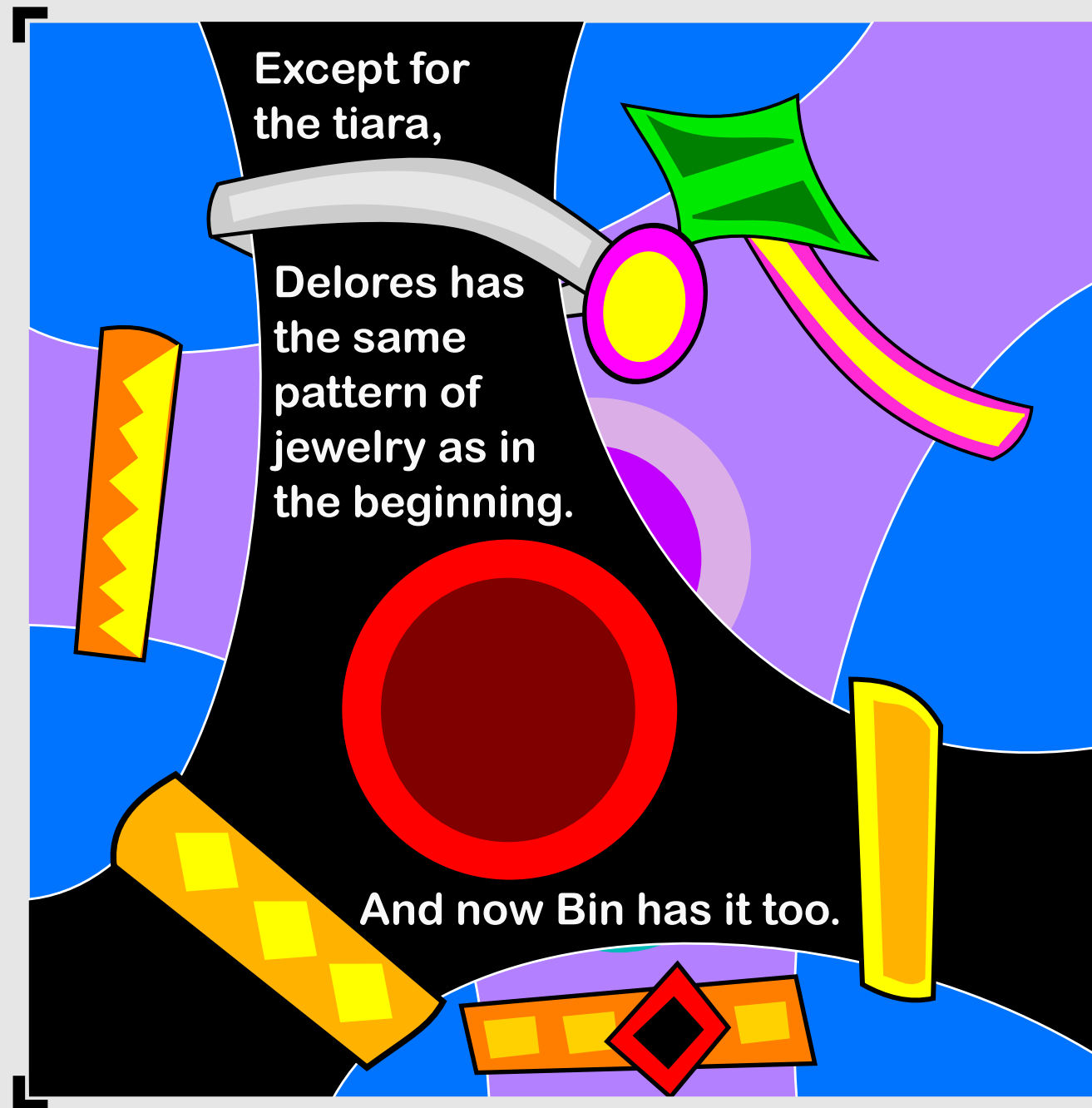


The fish move one piece of each pair over to Bin...

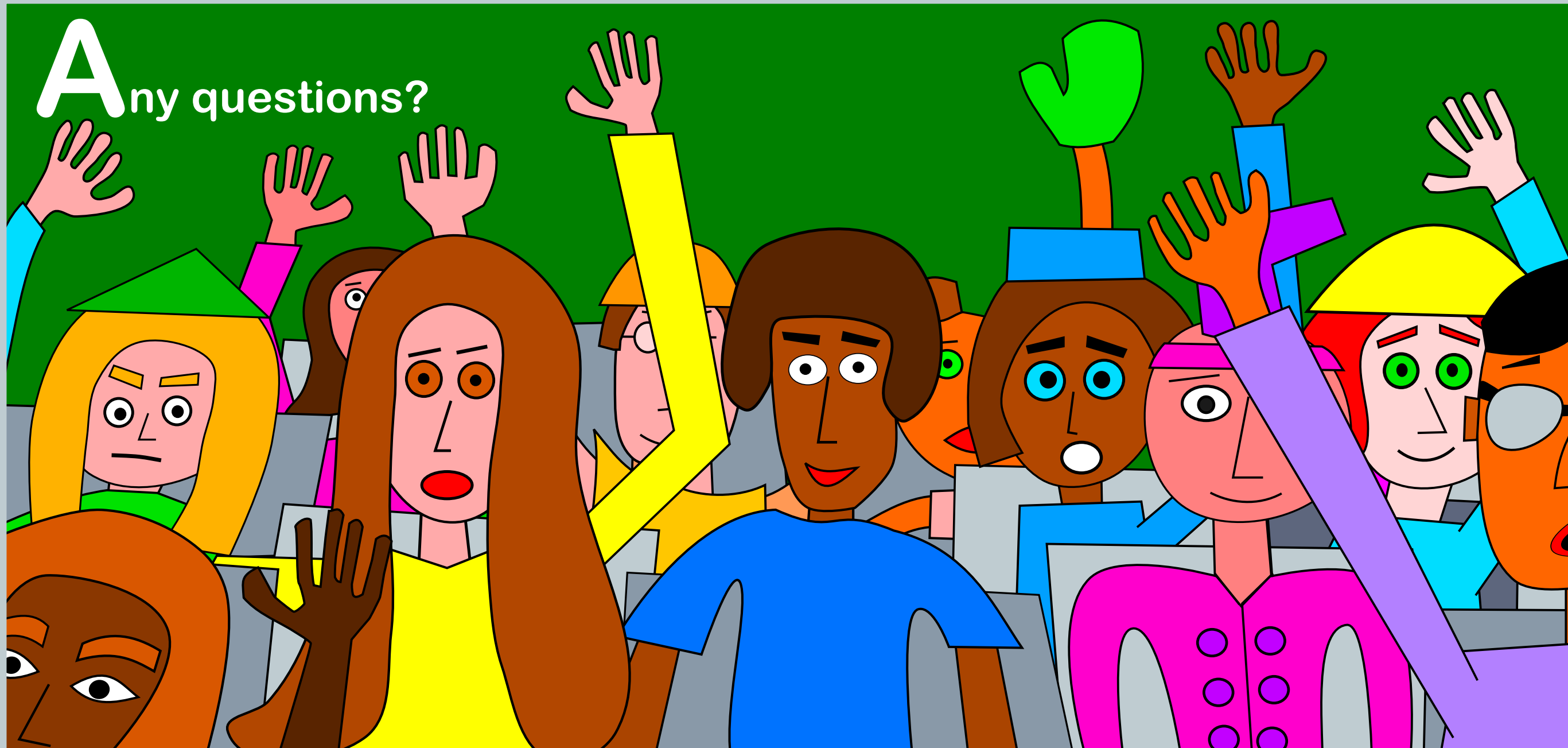
A vibrant, abstract illustration featuring a large, stylized fish with a yellow body and a blue eye, positioned in the upper right. The fish is surrounded by various geometric shapes and colors, including a large red circle, a green triangle, and a yellow rectangle. A white arrow points from the text "The fish move one piece of each pair over to Bin..." towards the fish. The background is a mix of blue, purple, and black, with a large, stylized fish shape in the center. The overall style is reminiscent of a children's book or a colorful poster.

A vibrant, stylized illustration. In the center is a large, bright purple sun with a lighter purple circular glow around it. To the right, a large rainbow fish with yellow, orange, and red stripes is swimming towards the right. Above it, a smaller green fish with blue stripes is also swimming right. In the bottom left, there's a green, leaf-like shape and a yellow and orange striped shape. The background is a mix of blue and purple. In the top right corner, the text "and then swim away." is written in a simple, black font.

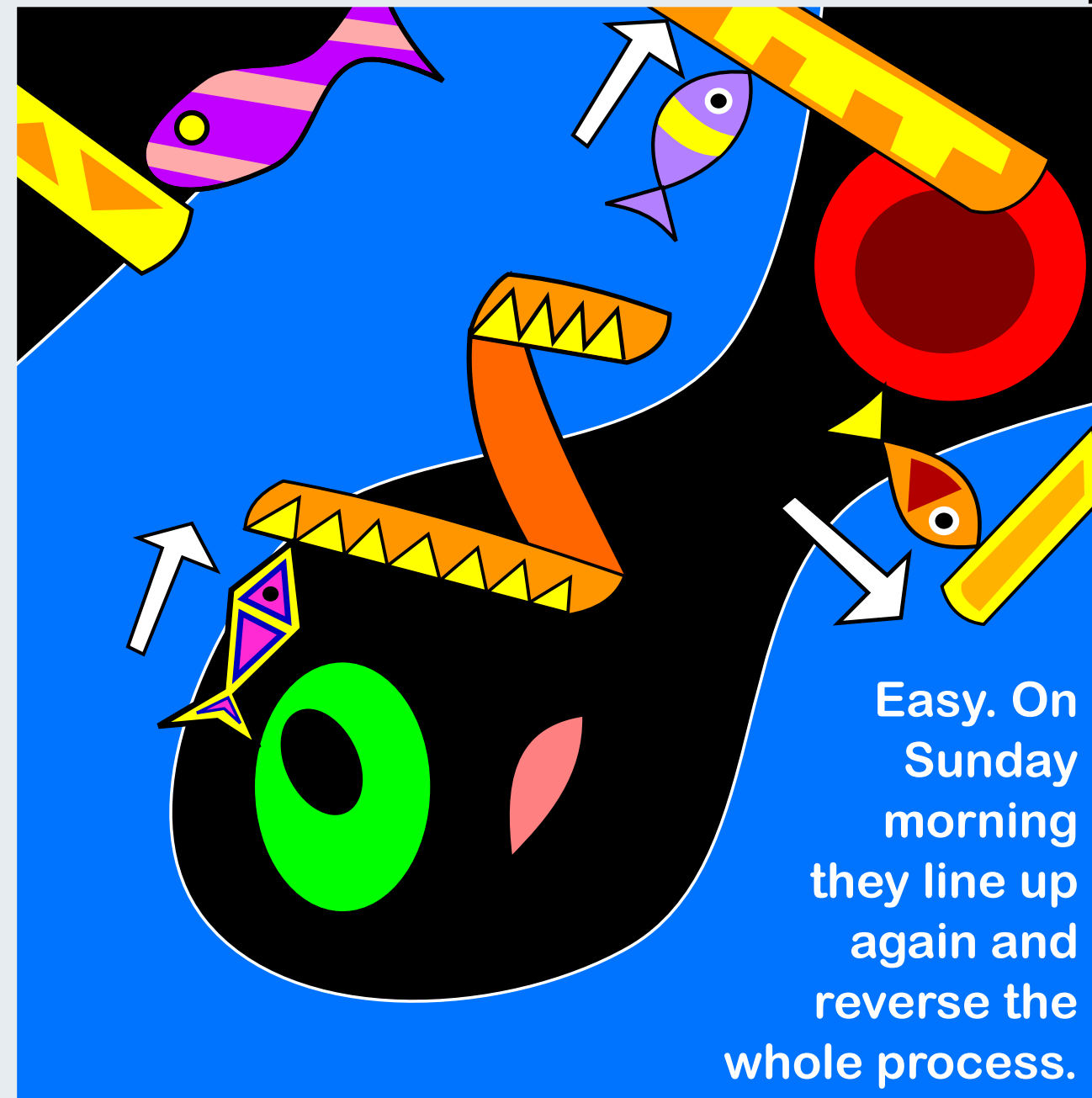




A ny questions?

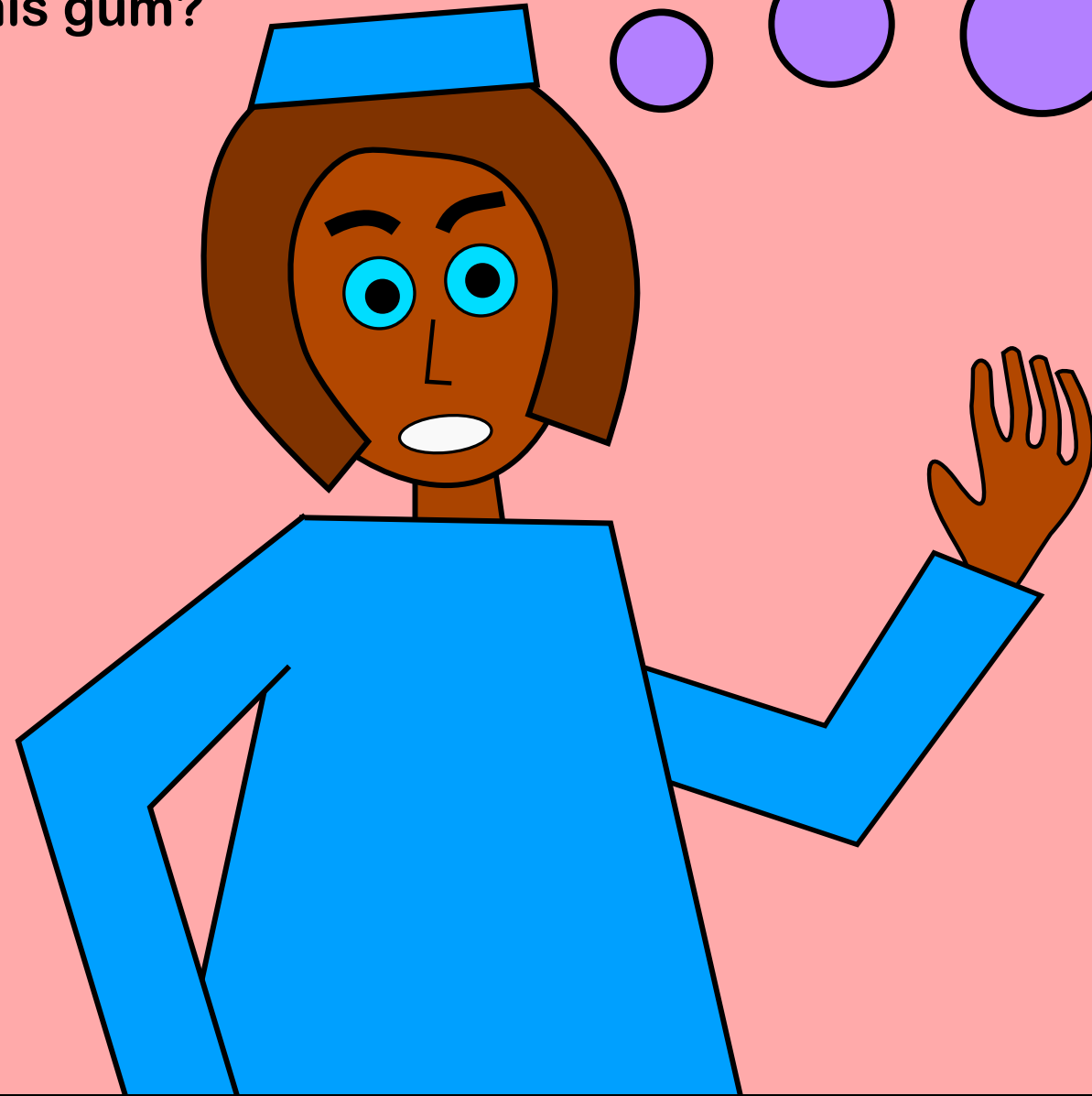


How does  
Delores get  
her jewelry  
back from  
Bin?

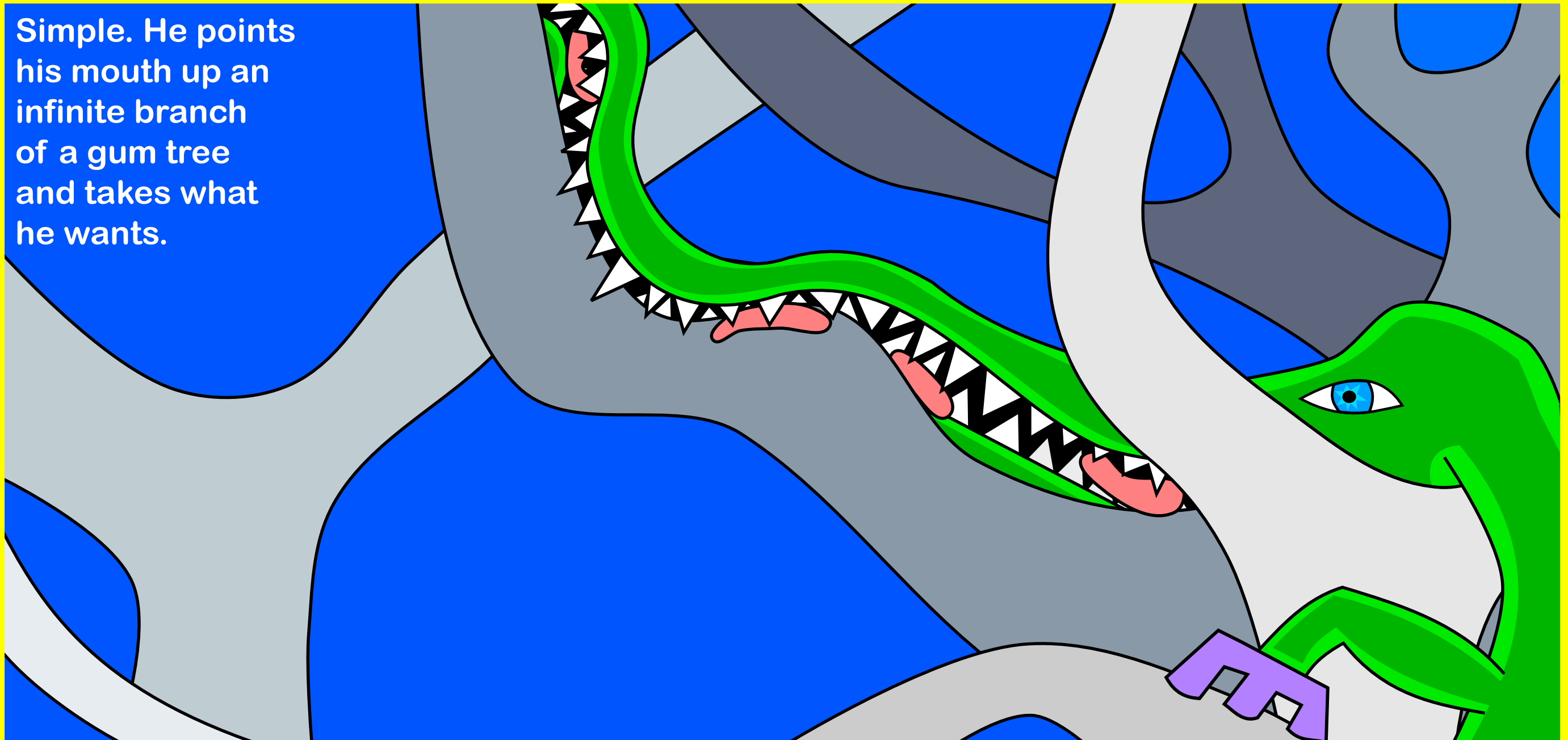


Easy. On  
Sunday  
morning  
they line up  
again and  
reverse the  
whole process.

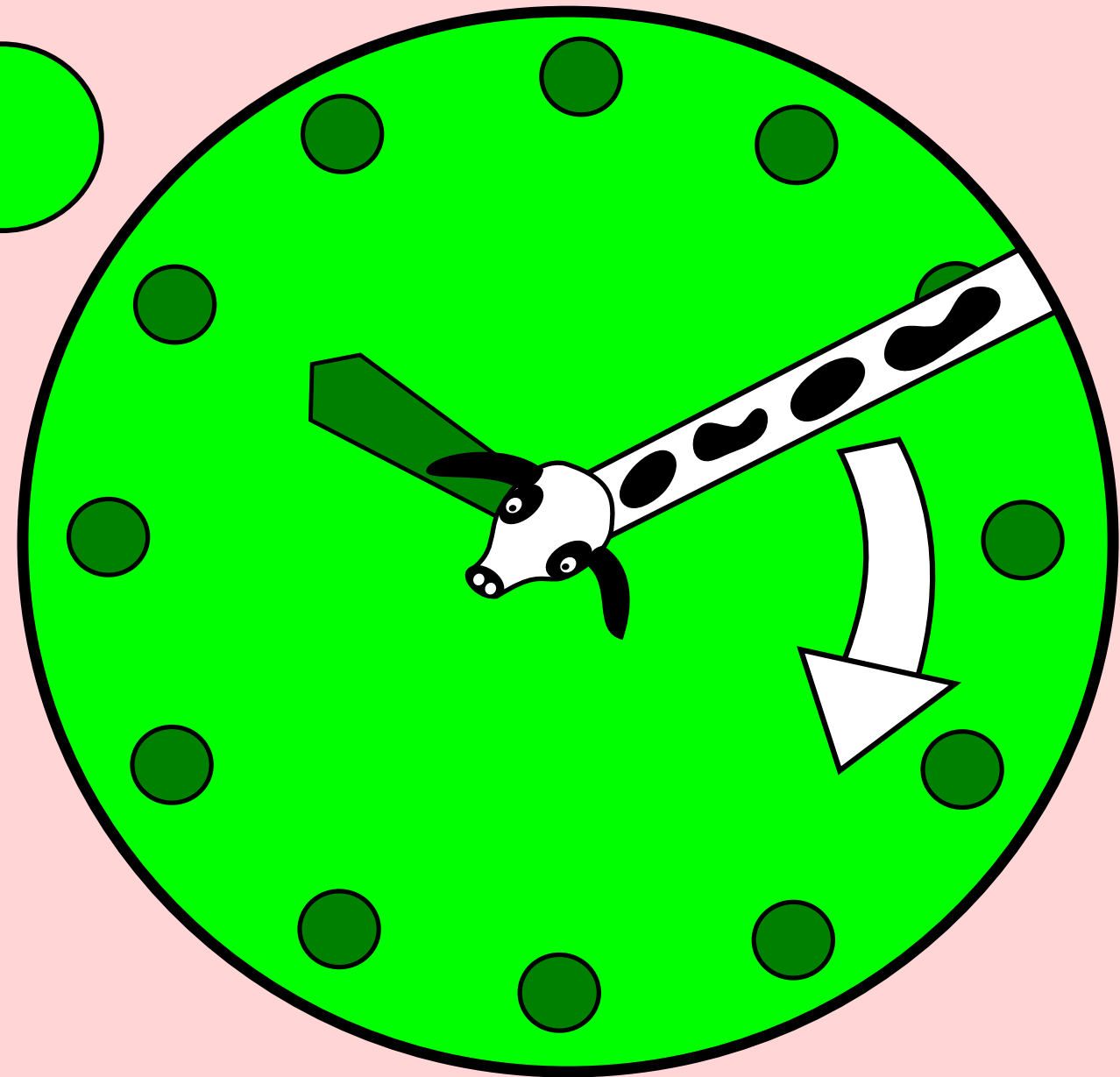
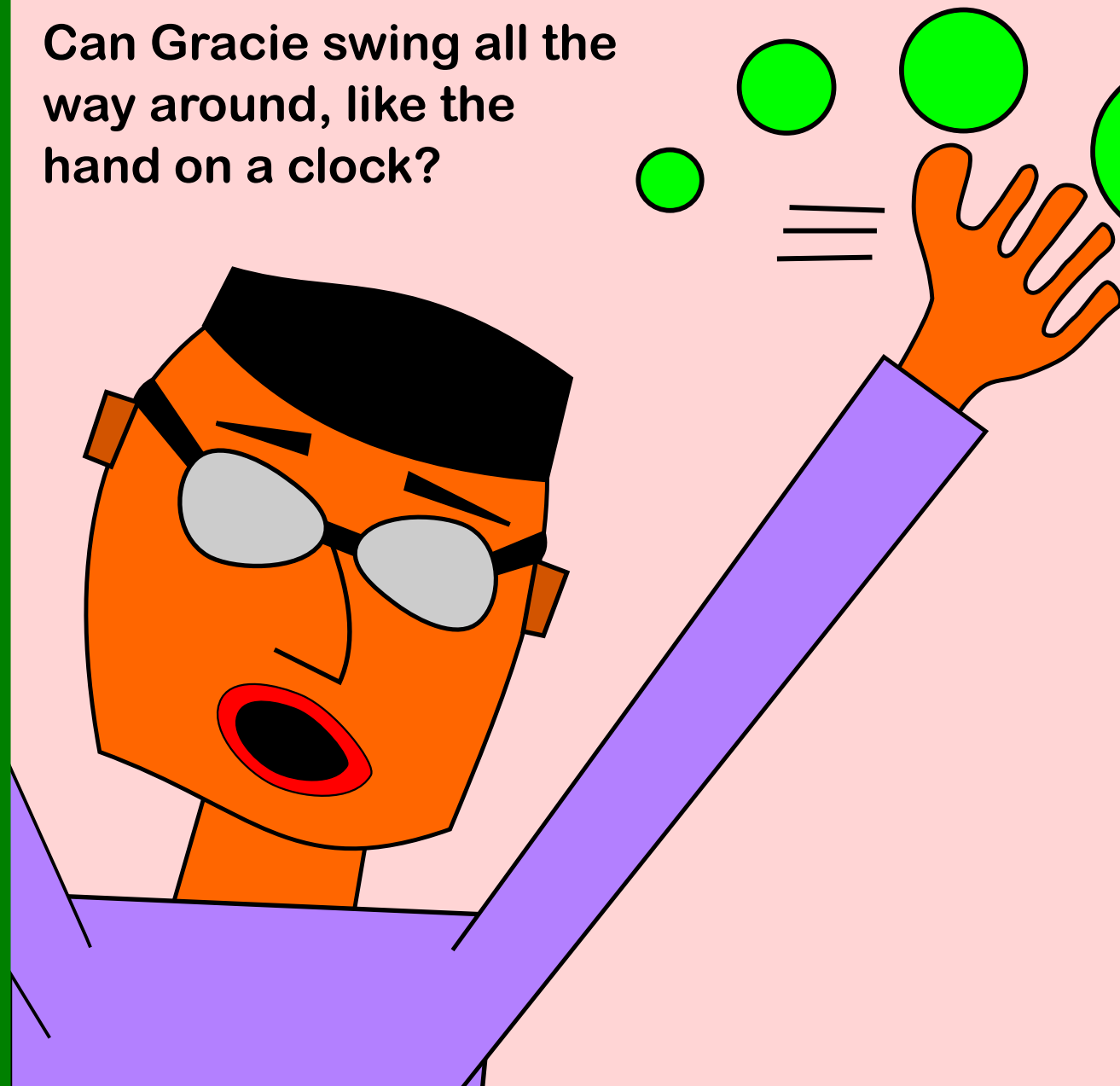
Where does  
Hammerwood  
get his gum?



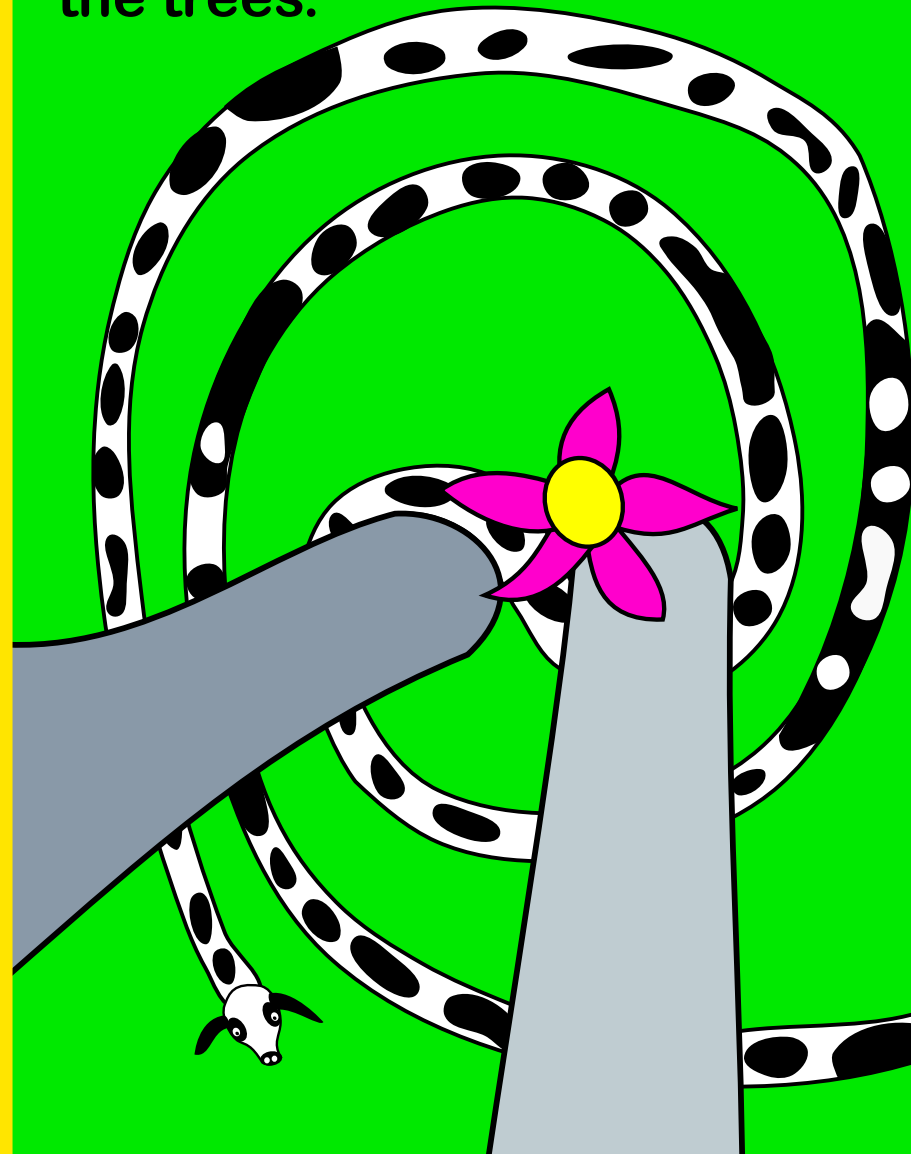
Simple. He points  
his mouth up an  
infinite branch  
of a gum tree  
and takes what  
he wants.



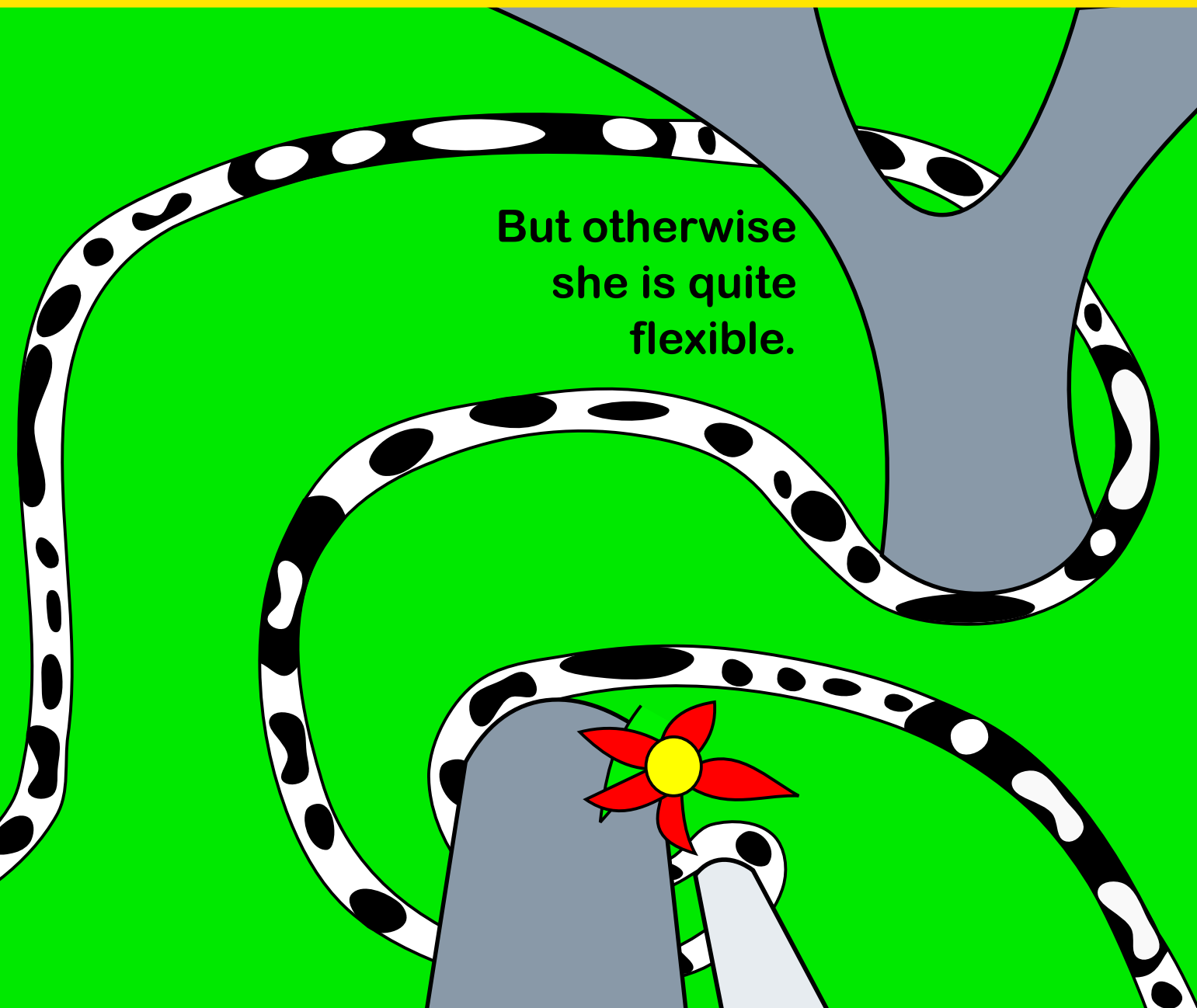
Can Gracie swing all the way around, like the hand on a clock?

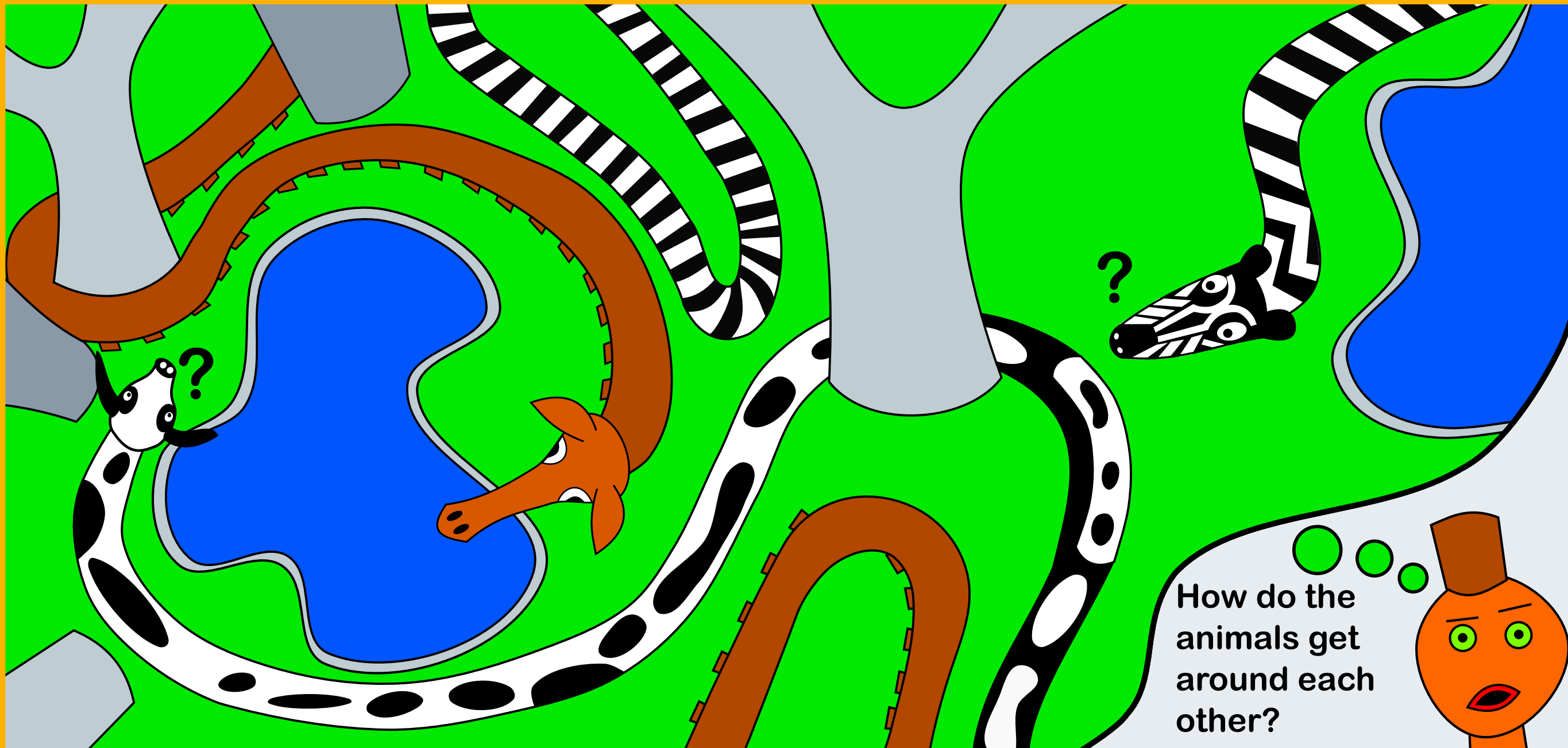


No. She would get stuck on the trees.

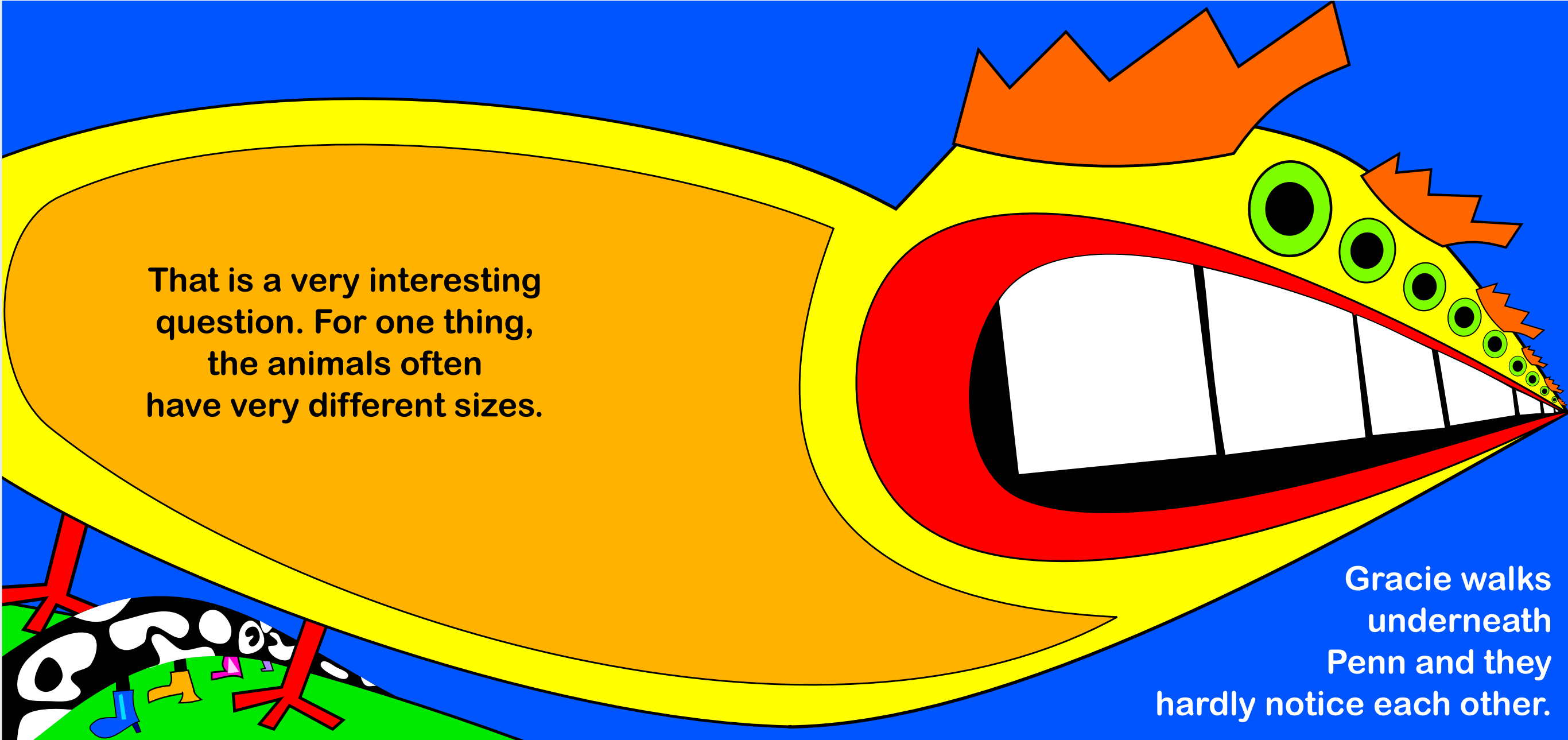


But otherwise  
she is quite  
flexible.





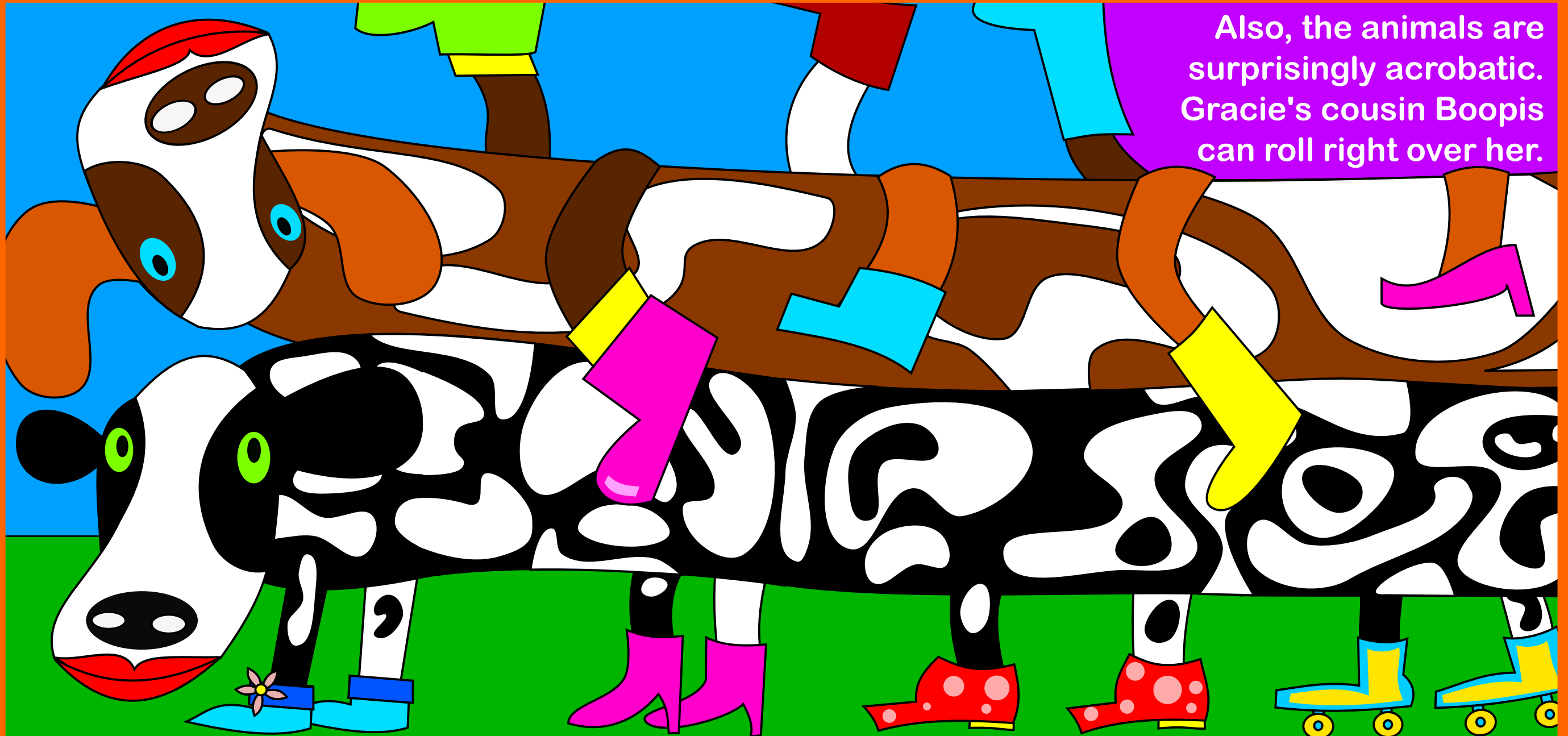




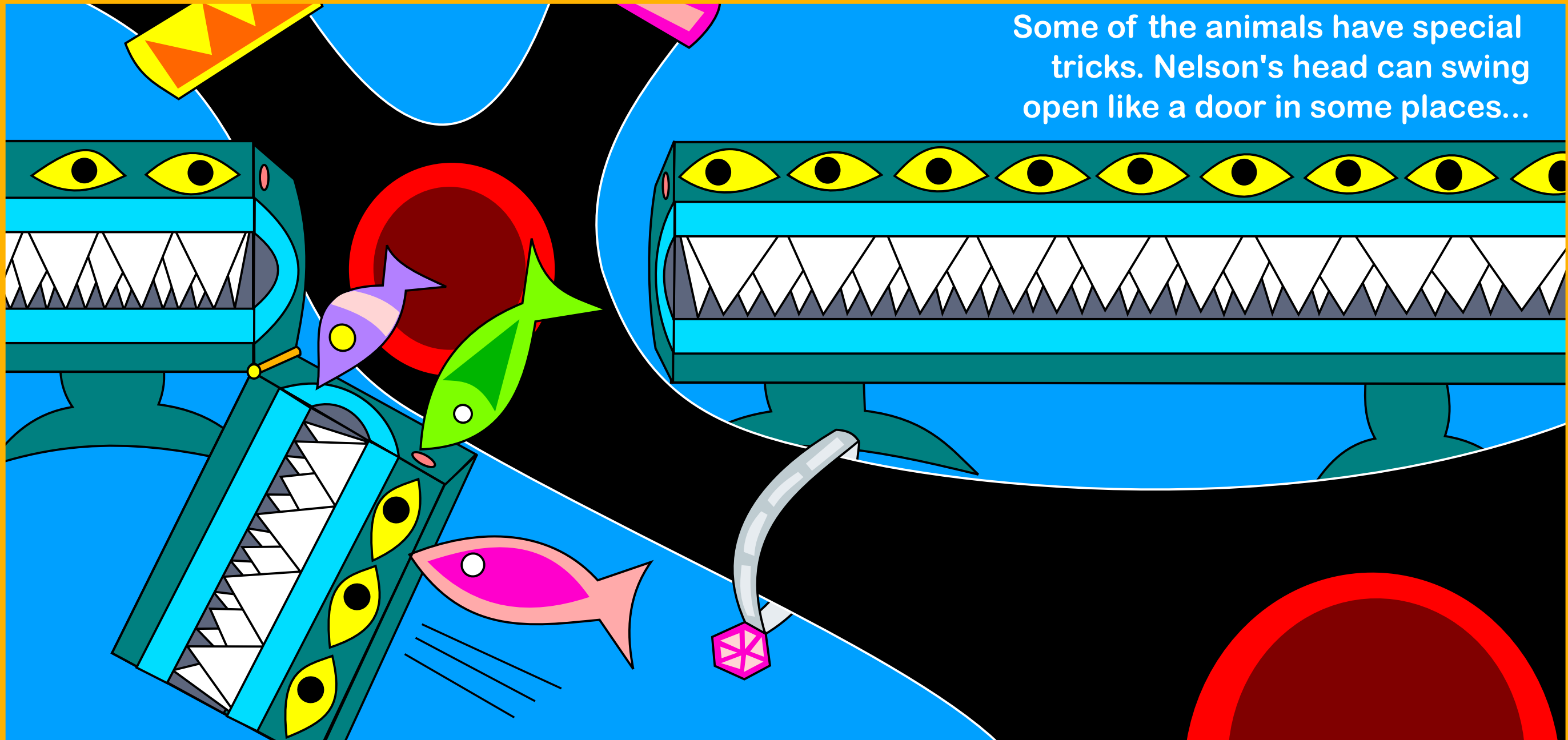
That is a very interesting  
question. For one thing,  
the animals often  
have very different sizes.

Gracie walks  
underneath  
Penn and they  
hardly notice each other.

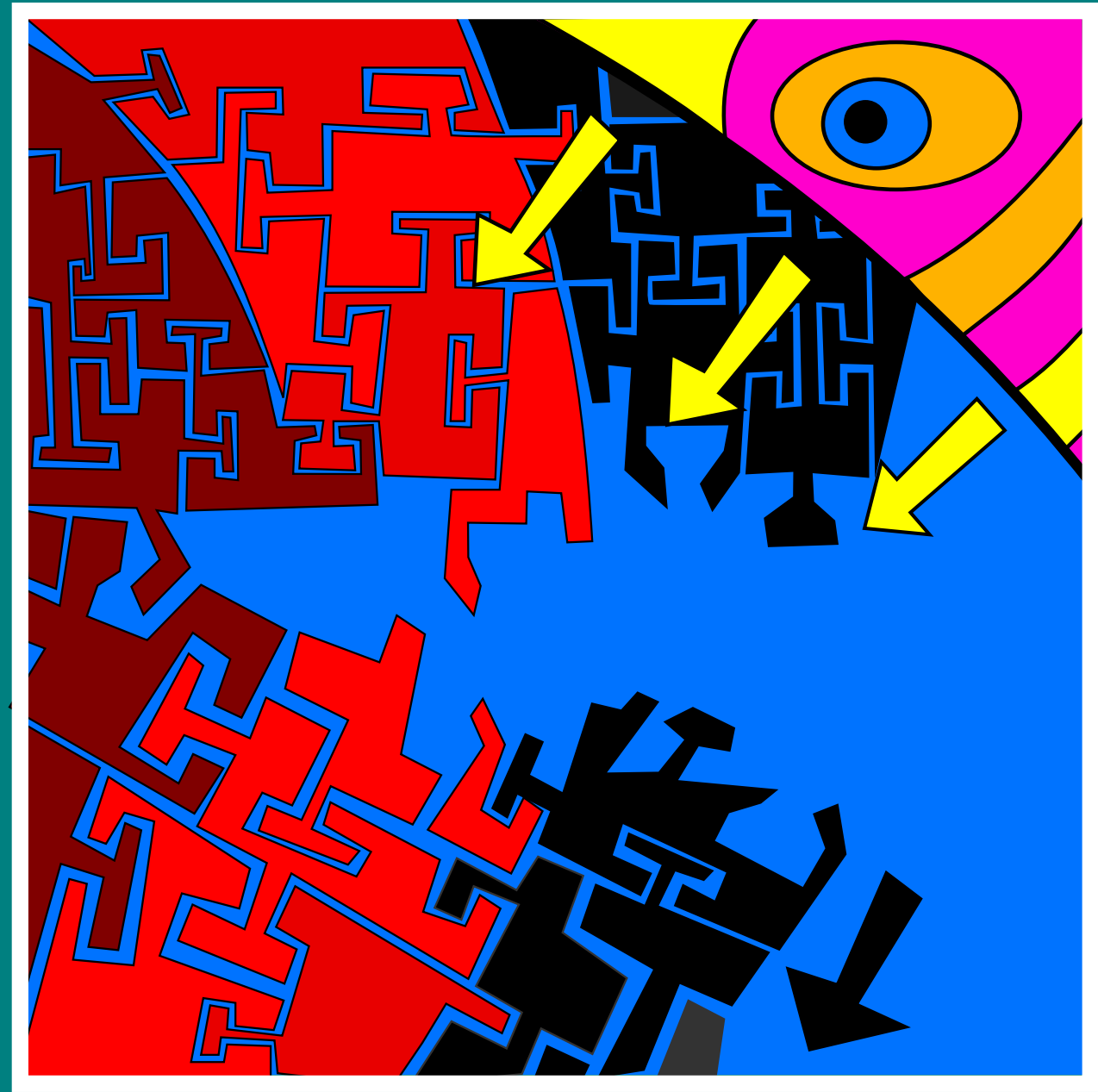
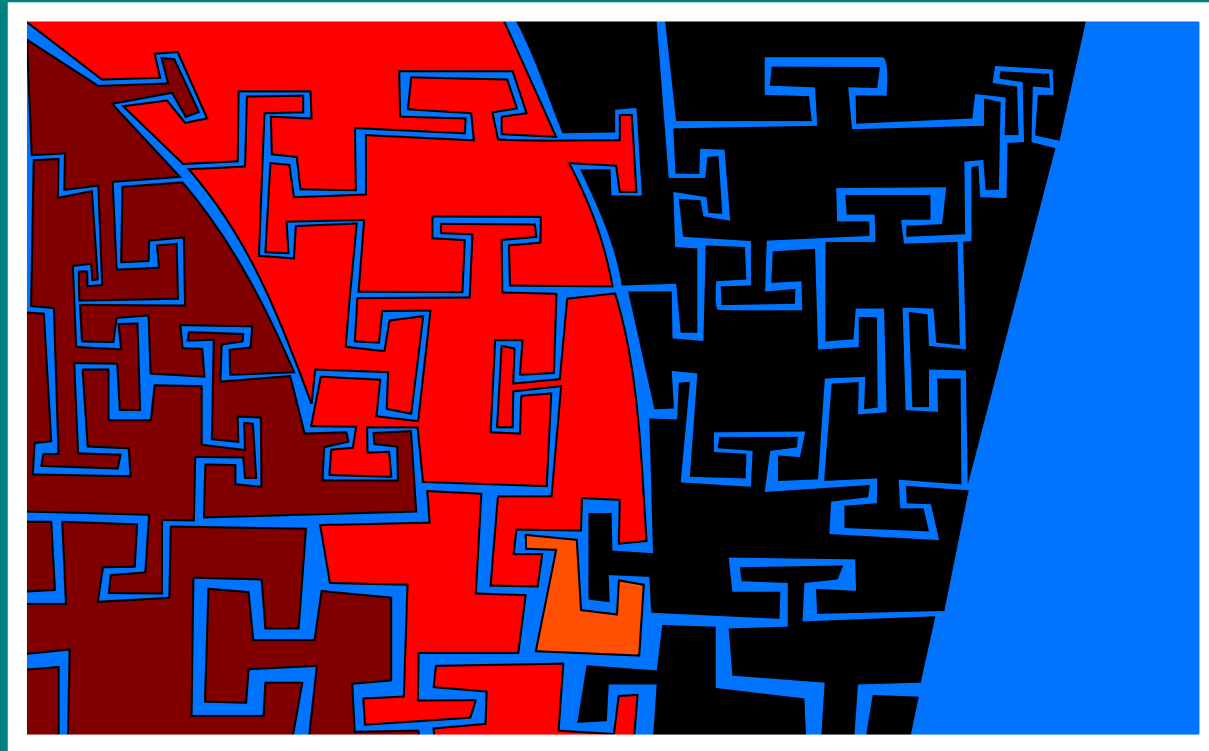
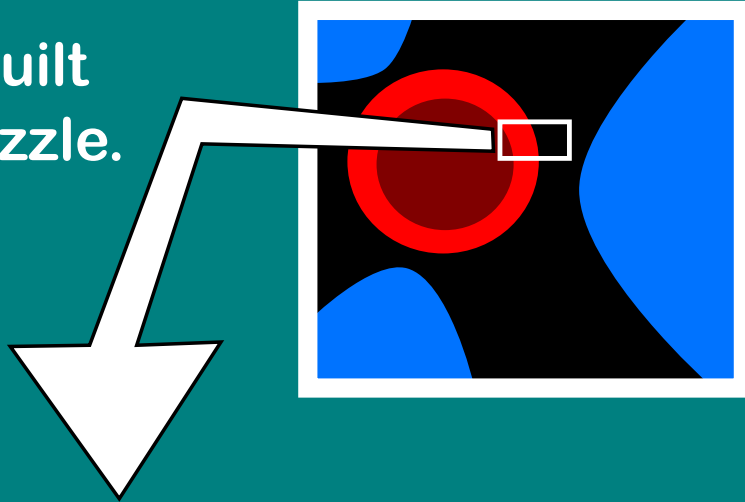
Also, the animals are surprisingly acrobatic. Gracie's cousin Boopis can roll right over her.

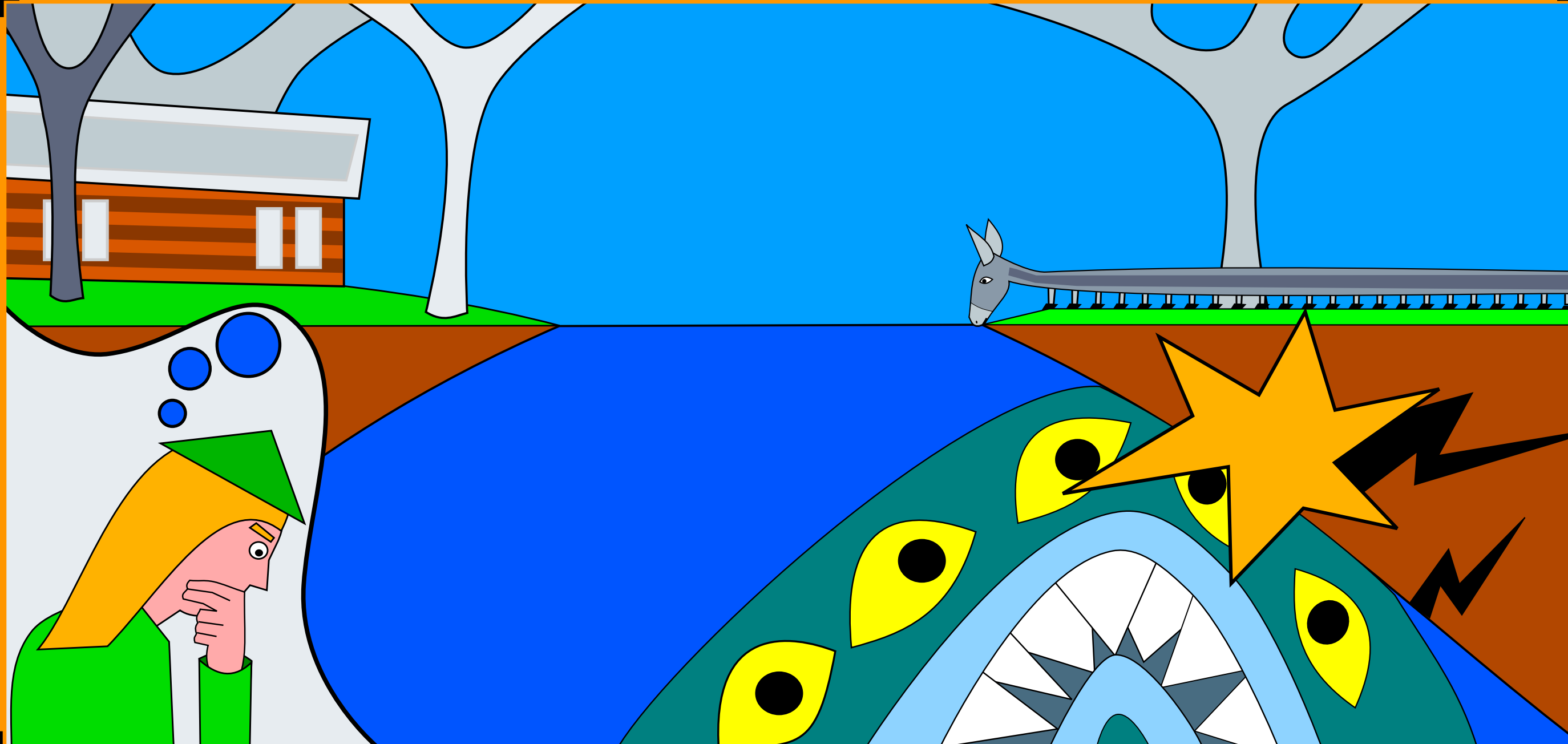


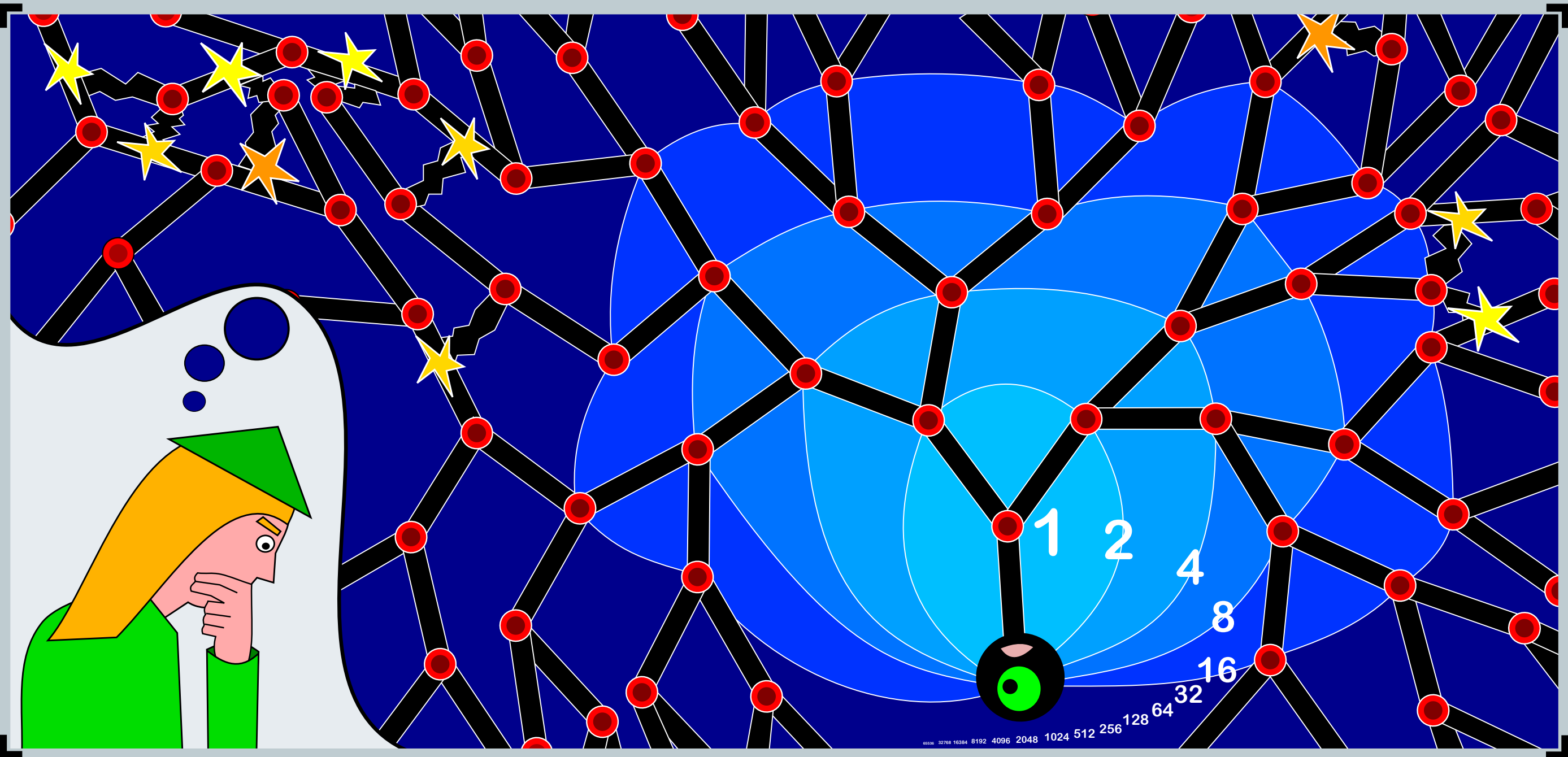
Some of the animals have special tricks. Nelson's head can swing open like a door in some places...



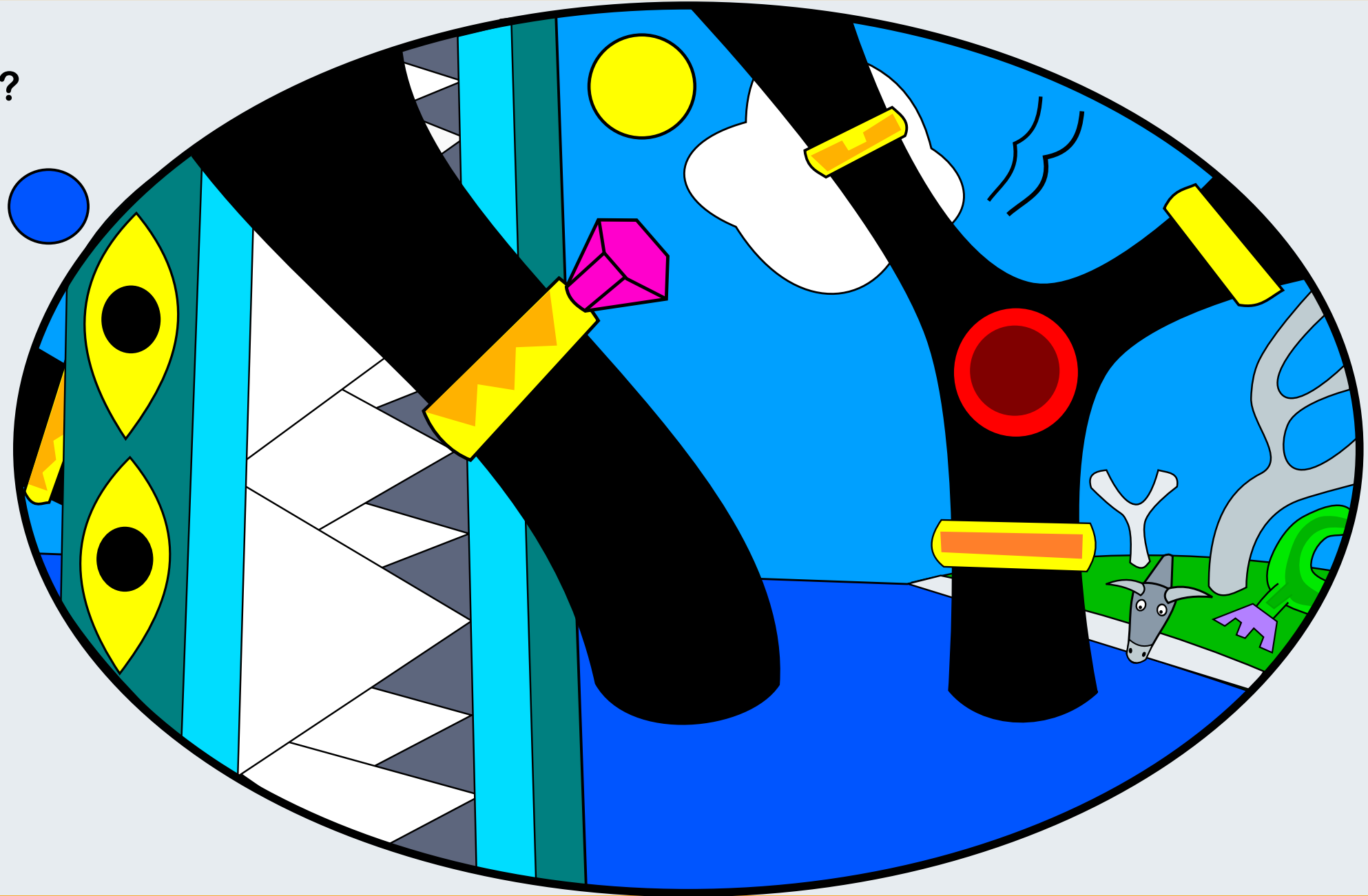
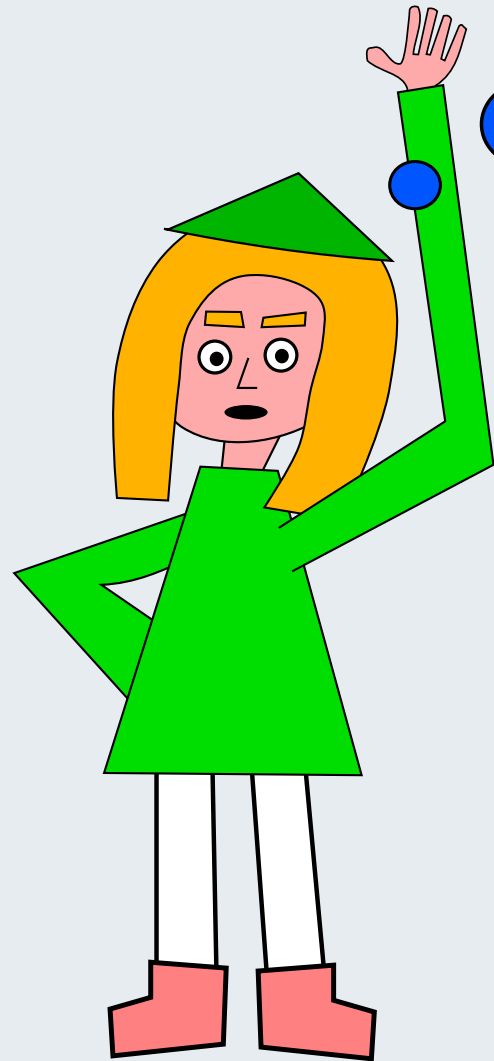
and Delores is built  
like a flexible puzzle.  
She can pull  
herself apart  
to let another  
animal through.



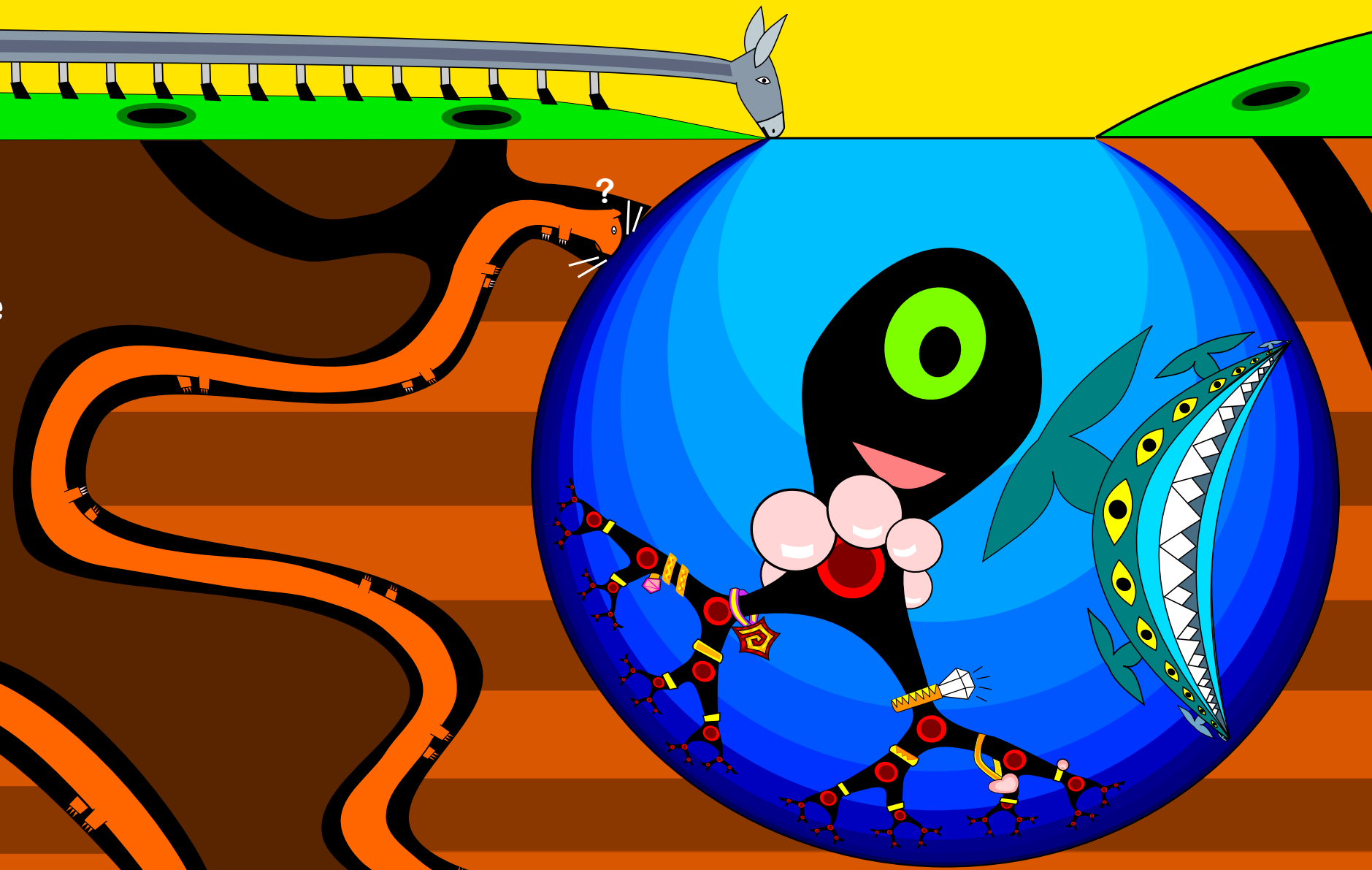




How do the infinite  
animals fit in the pond?

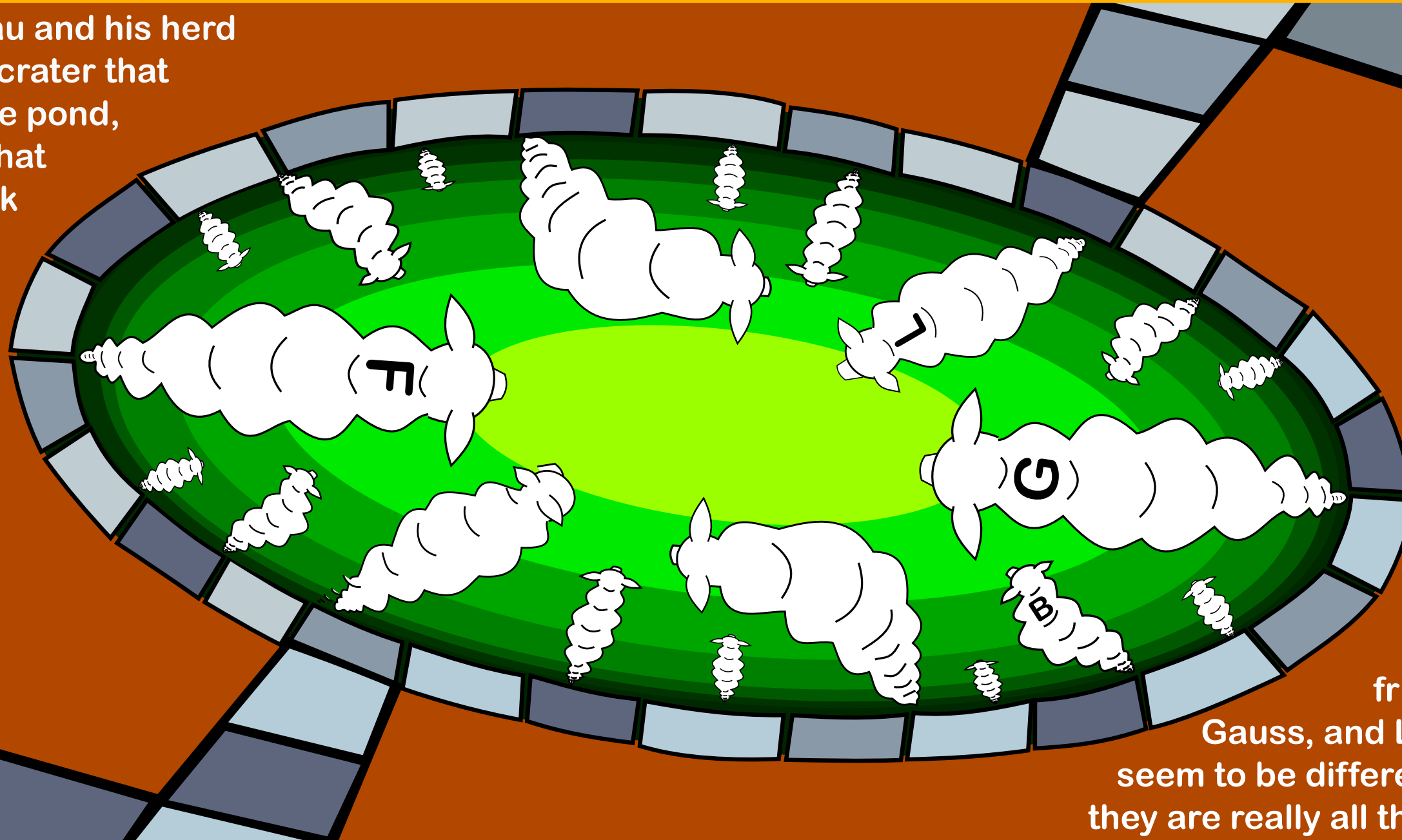


Like this! The space inside the pond is warped in a way that makes it infinite from the inside but finite from the outside. To us outsiders, Delores' spots appear to shrink as they approach the edge, but they are really all the same size. The same goes for Nelson's teeth and eyes.

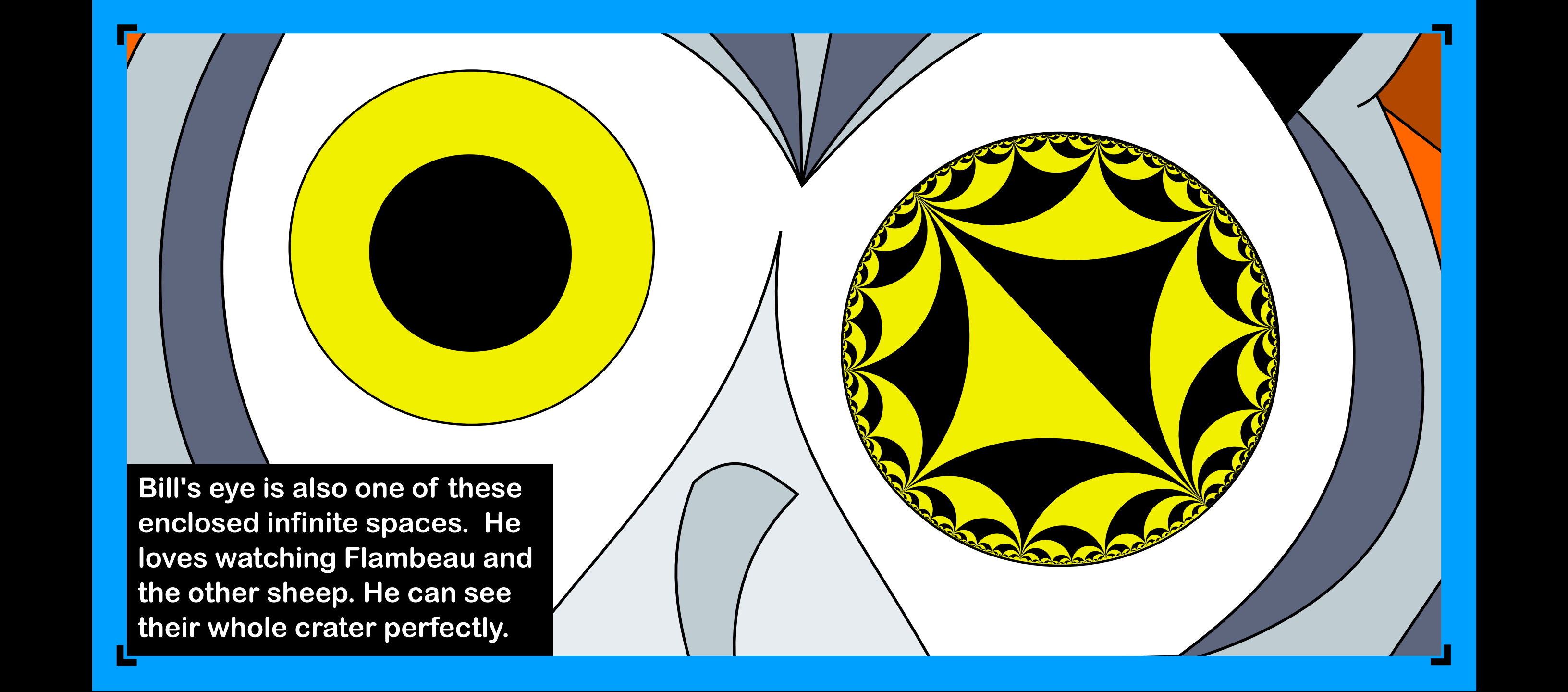




Flambeau and his herd  
live in a crater that  
is like the pond,  
except that  
they walk  
on the bottom.

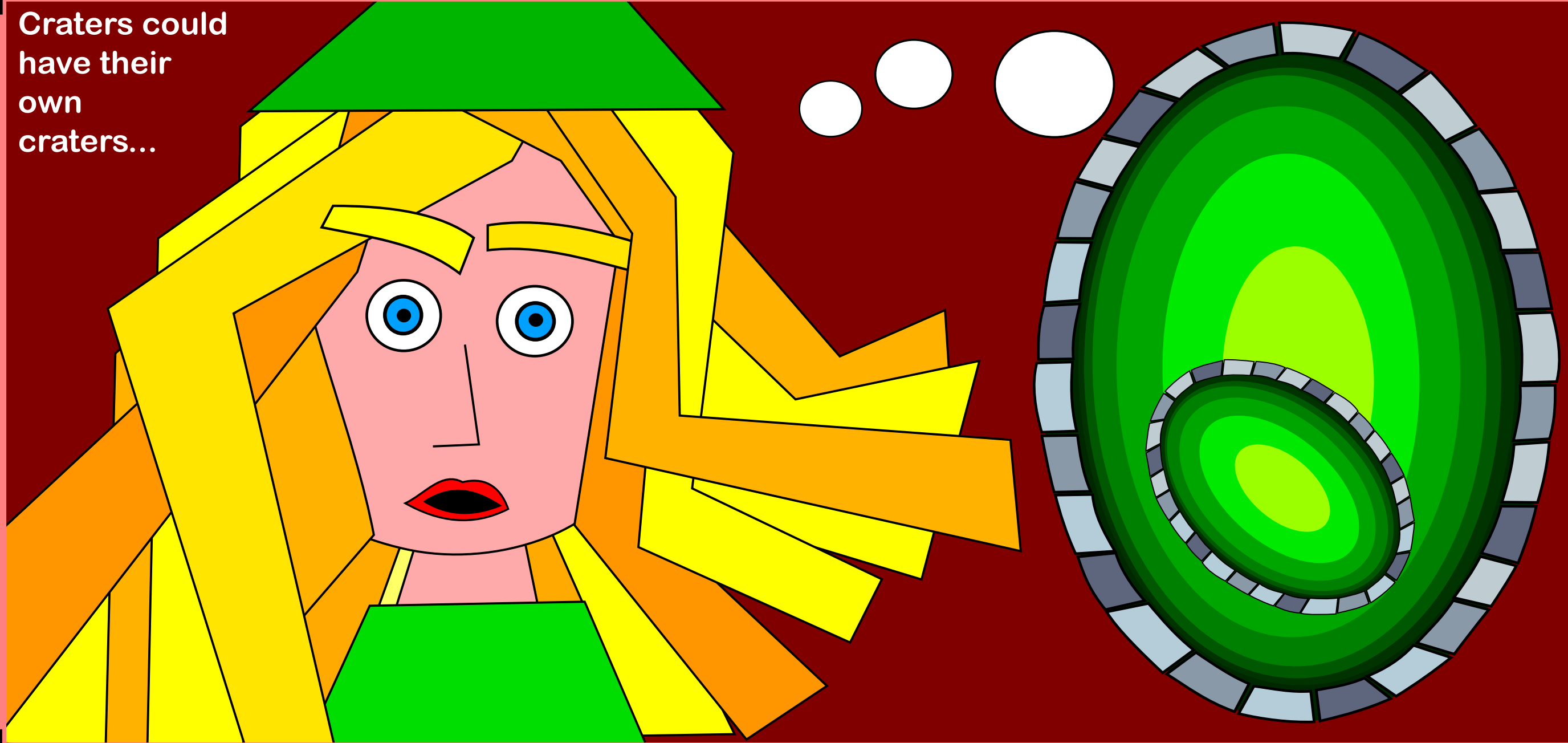


From the  
outside,  
Flambeau's  
friends Bolyai,  
Gauss, and Lobachevsky  
seem to be different sizes, but  
they are really all the same size.

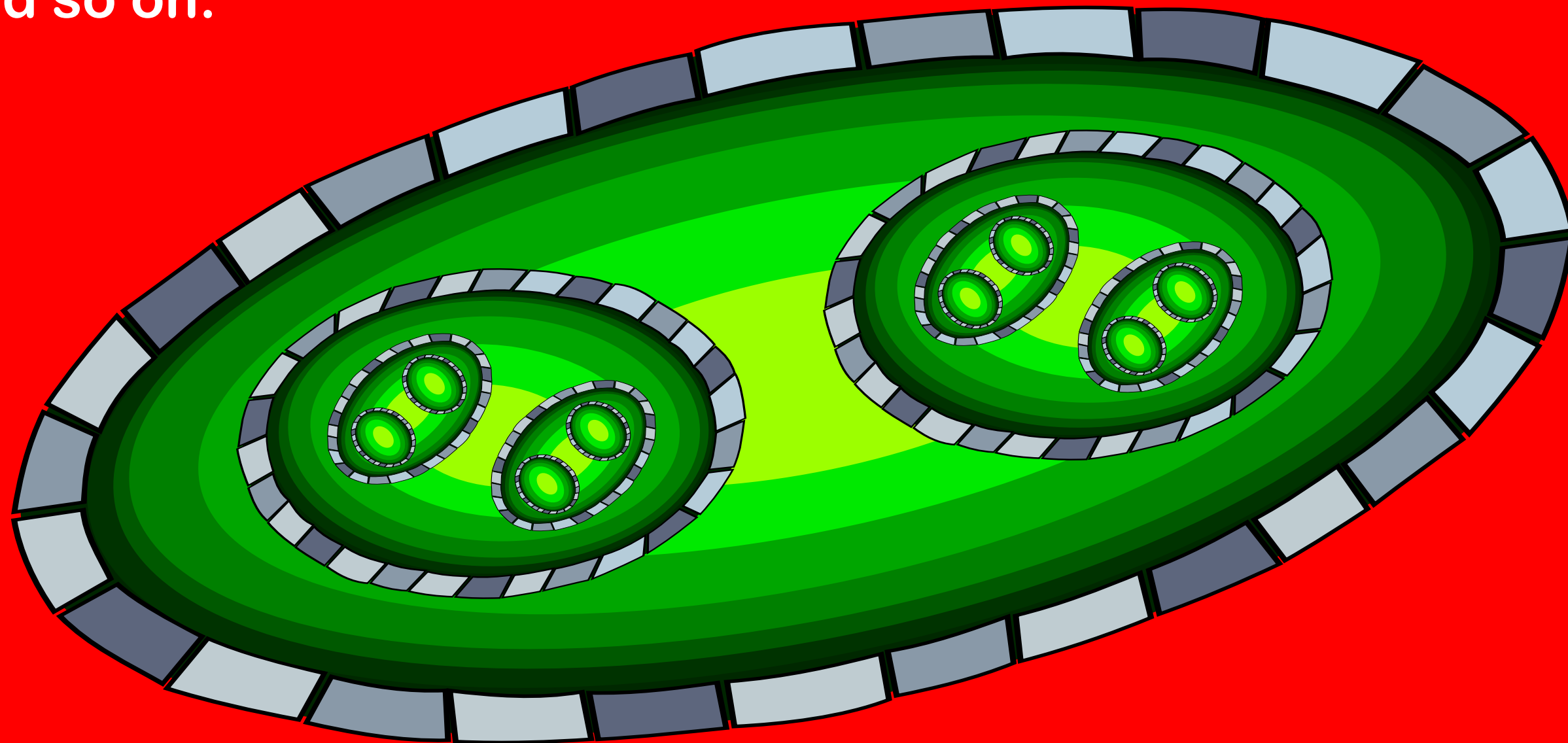


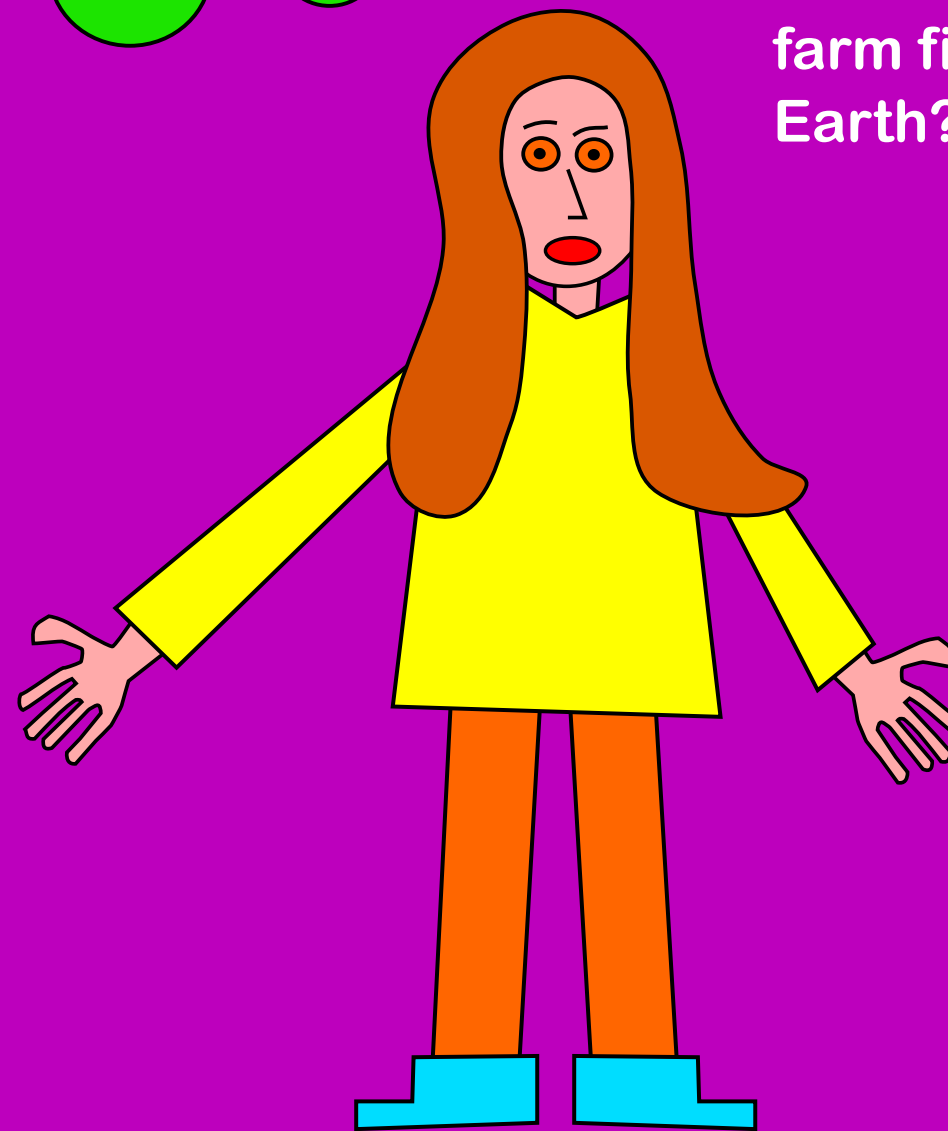
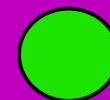
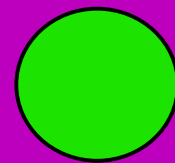
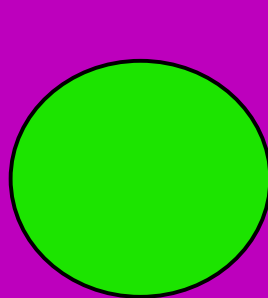
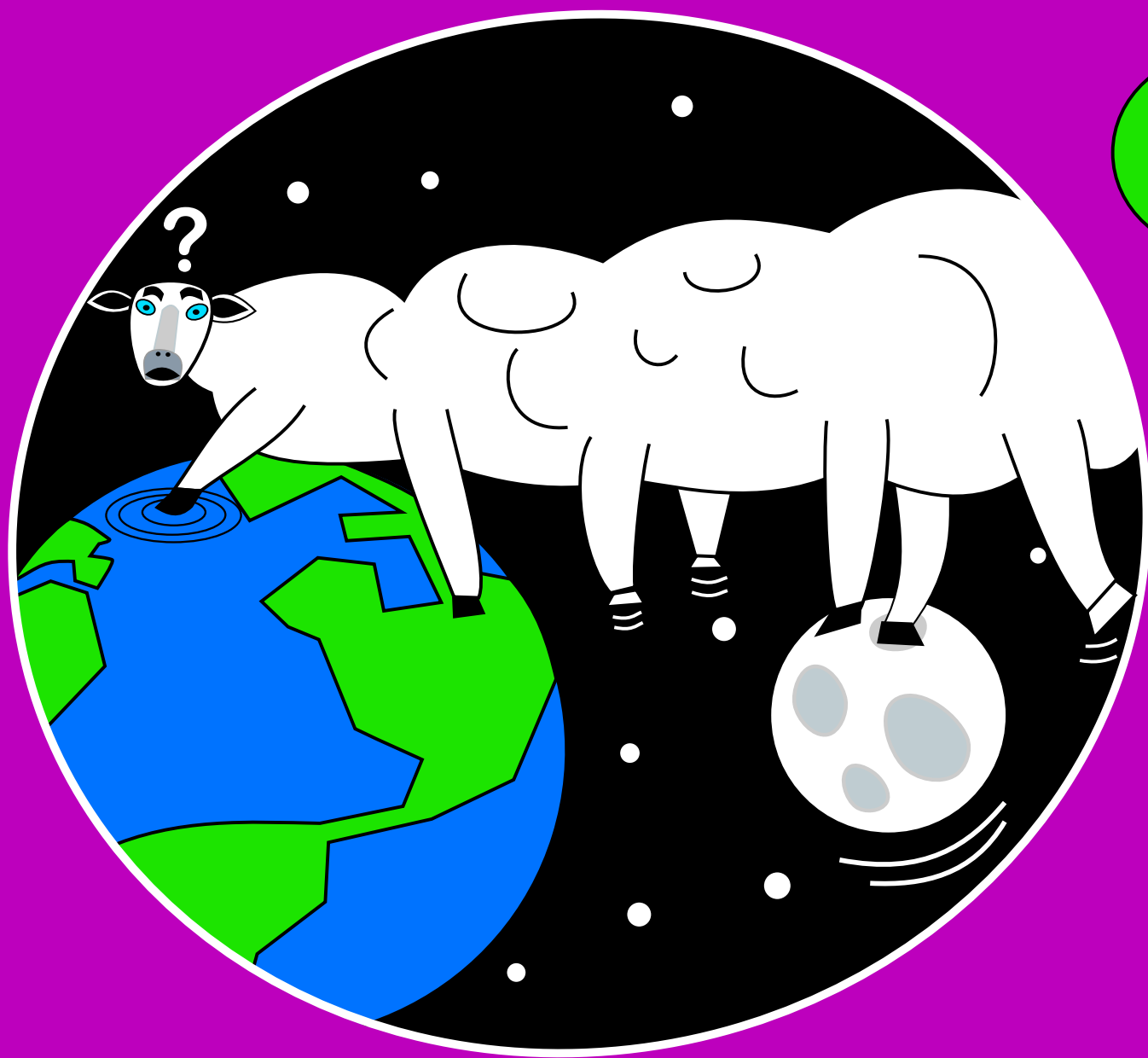
Bill's eye is also one of these enclosed infinite spaces. He loves watching Flambeau and the other sheep. He can see their whole crater perfectly.

Craters could  
have their  
own  
craters...



And so on.

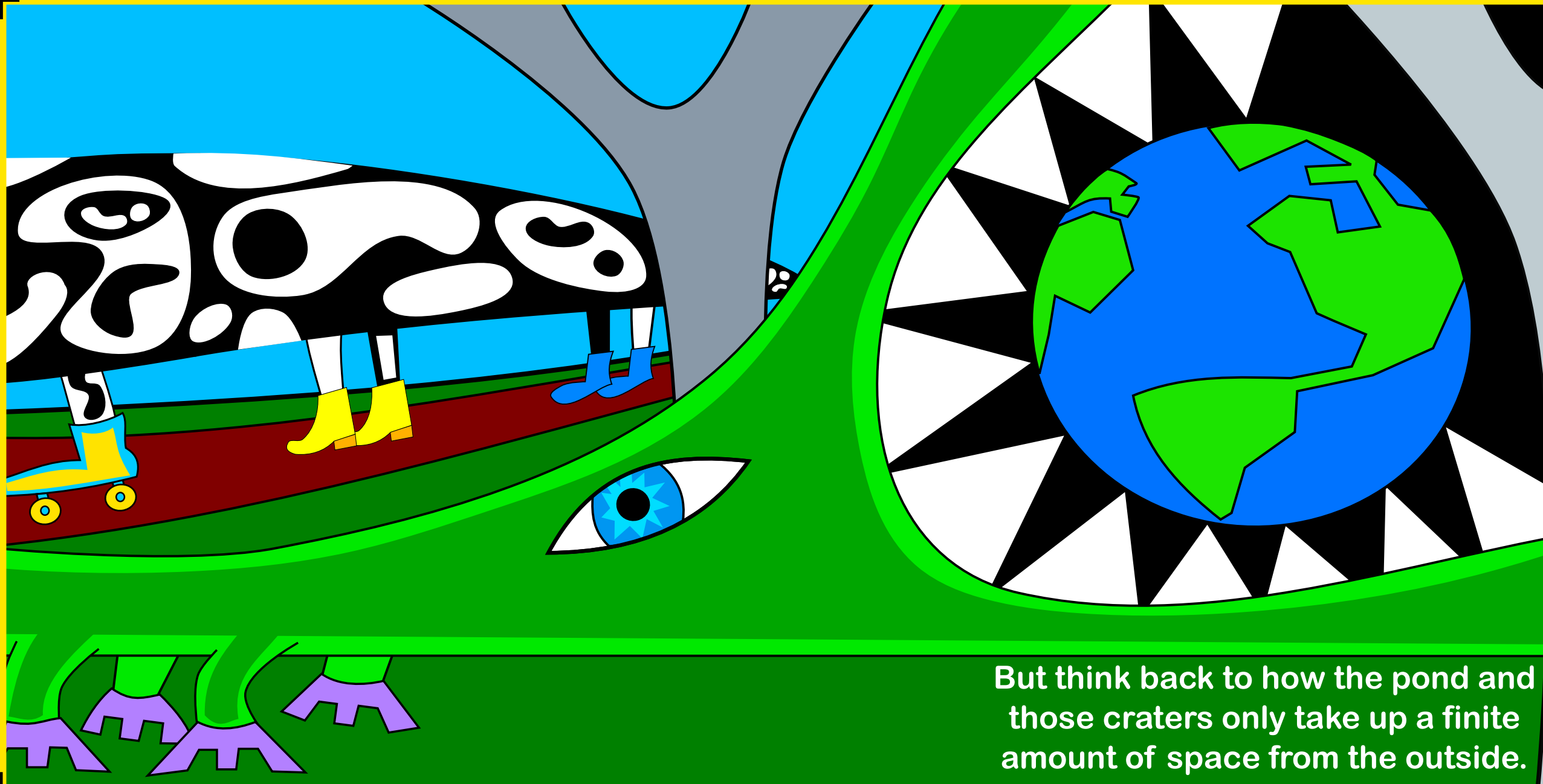




Can the  
infinite  
farm fit on  
Earth?



The infinite farm doesn't  
close up like Earth, so  
how could it fit on Earth?  
It seems impossible.

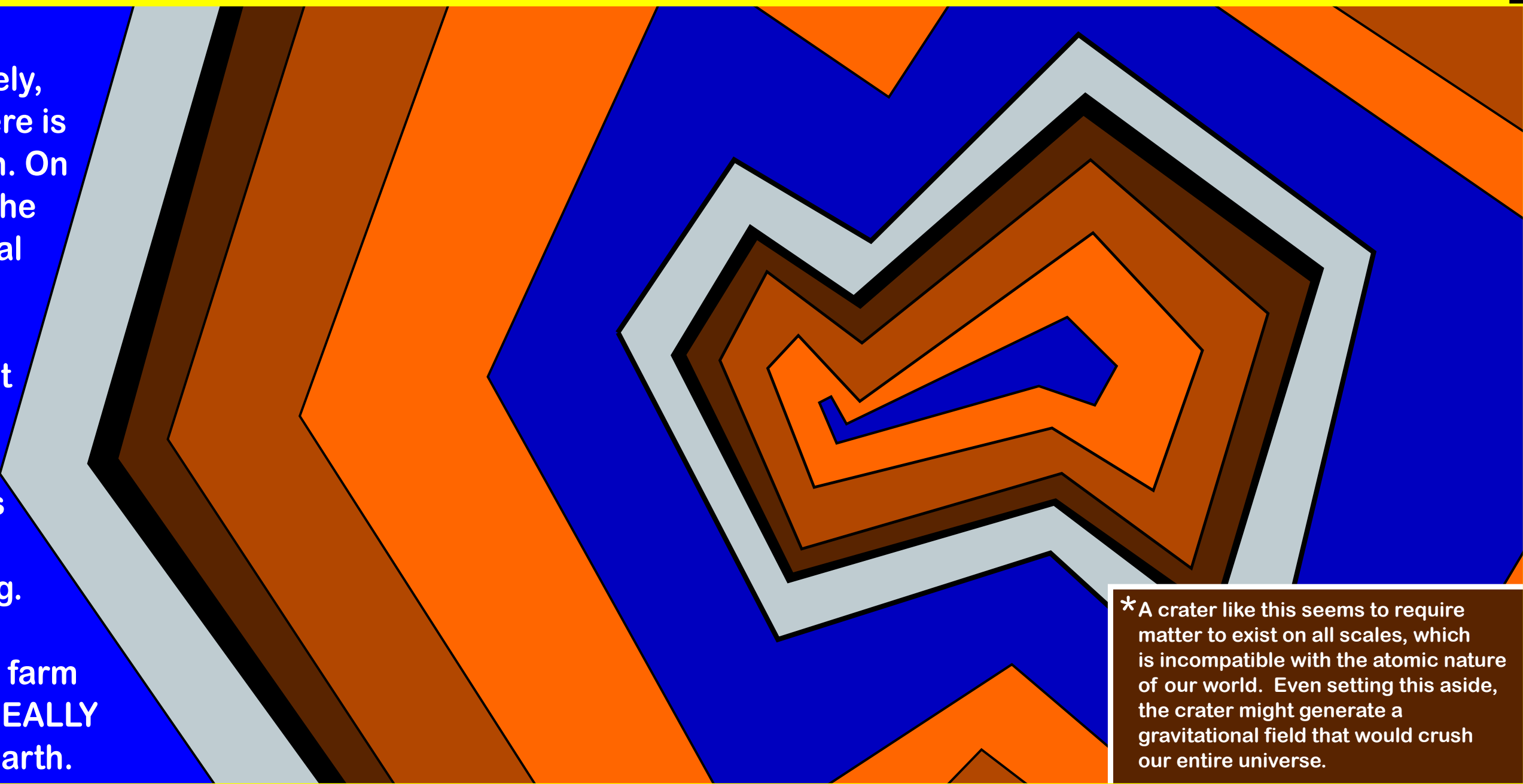


But think back to how the pond and those craters only take up a finite amount of space from the outside.



Oh, I see. It isn't such a stretch  
to imagine that the farm itself  
is one of those craters,  
and that the  
crater is on  
Earth.

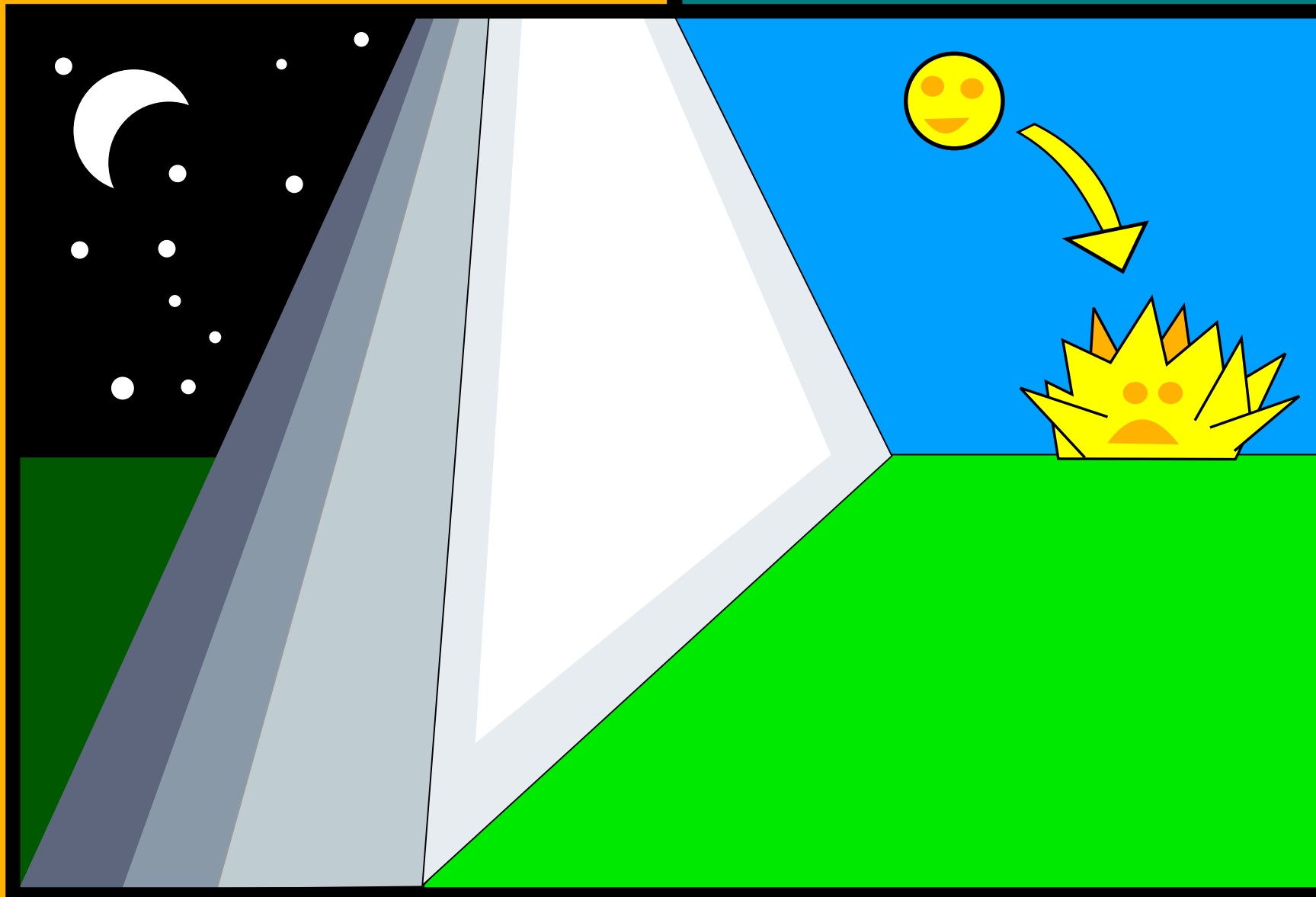




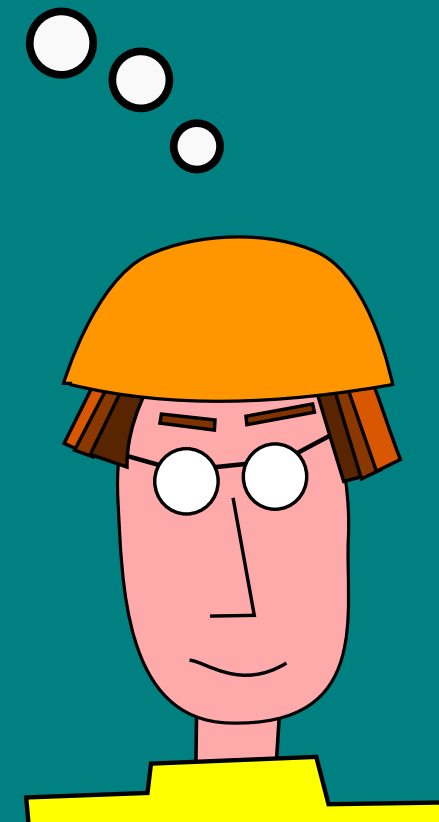
Yes,  
precisely,  
but there is  
a catch. On  
Earth the  
physical  
laws\*  
would  
prevent  
one of  
those  
craters  
from  
existing.  
So, the  
infinite farm  
can't REALLY  
fit on Earth.

\*A crater like this seems to require matter to exist on all scales, which is incompatible with the atomic nature of our world. Even setting this aside, the crater might generate a gravitational field that would crush our entire universe.

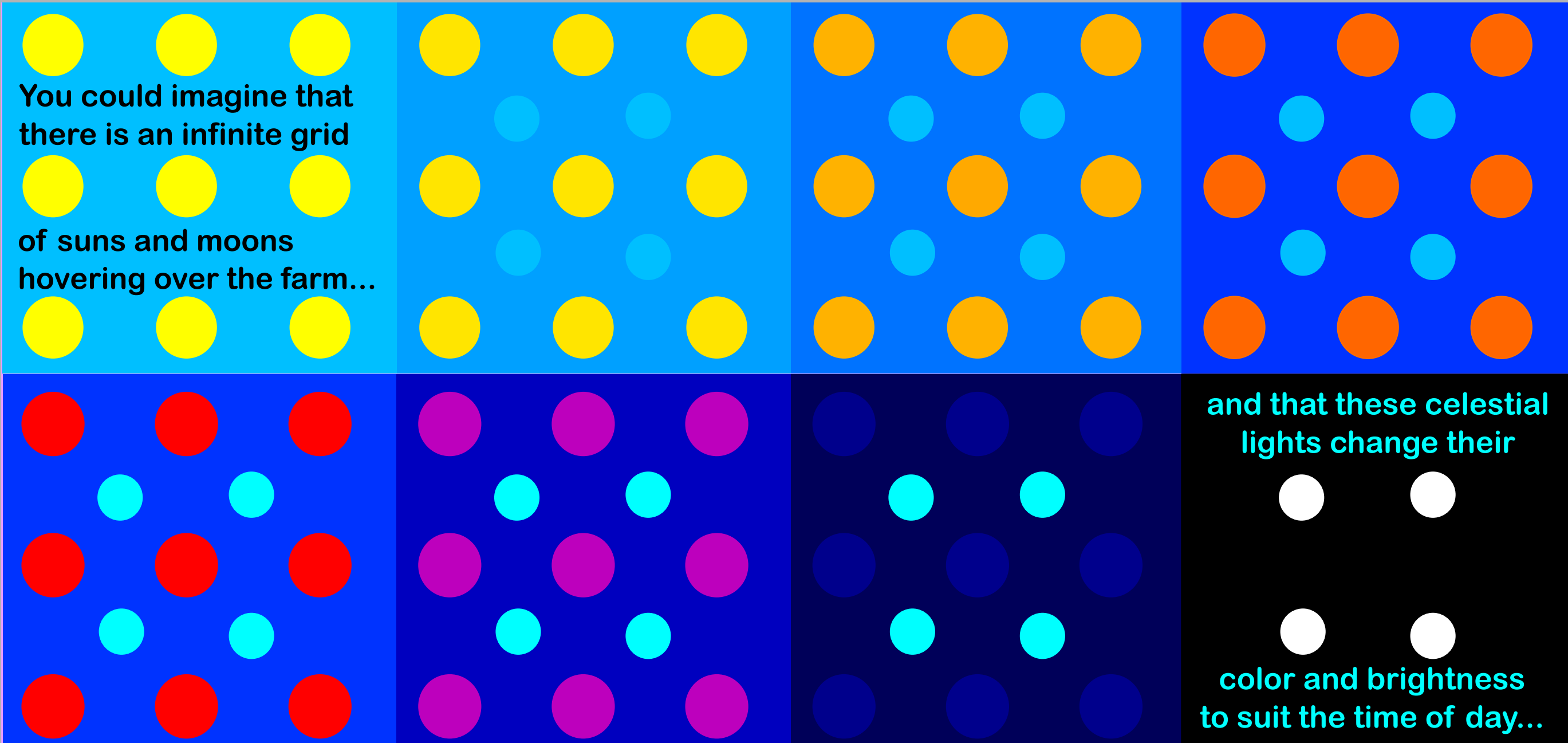
How does  
day change  
to night on  
the farm?



Yeah, how  
does the  
sun get  
around  
the farm?

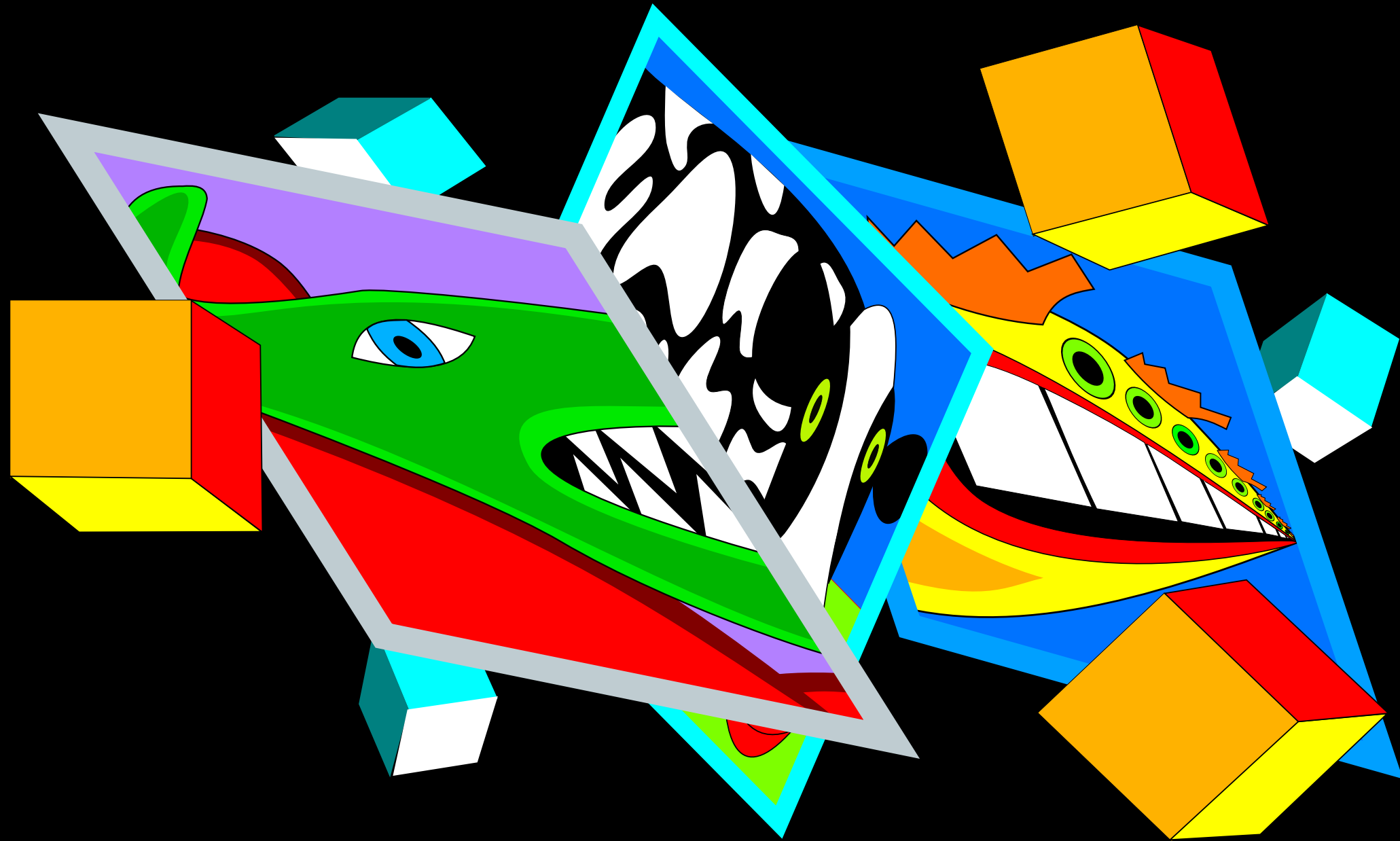


You could imagine that  
there is an infinite grid  
of suns and moons  
hovering over the farm...



and that these celestial  
lights change their  
color and brightness  
to suit the time of day...

or you could  
imagine that  
the suns and  
moons dance  
around the  
farm in a  
complicated,  
multi-dimensional  
pattern...



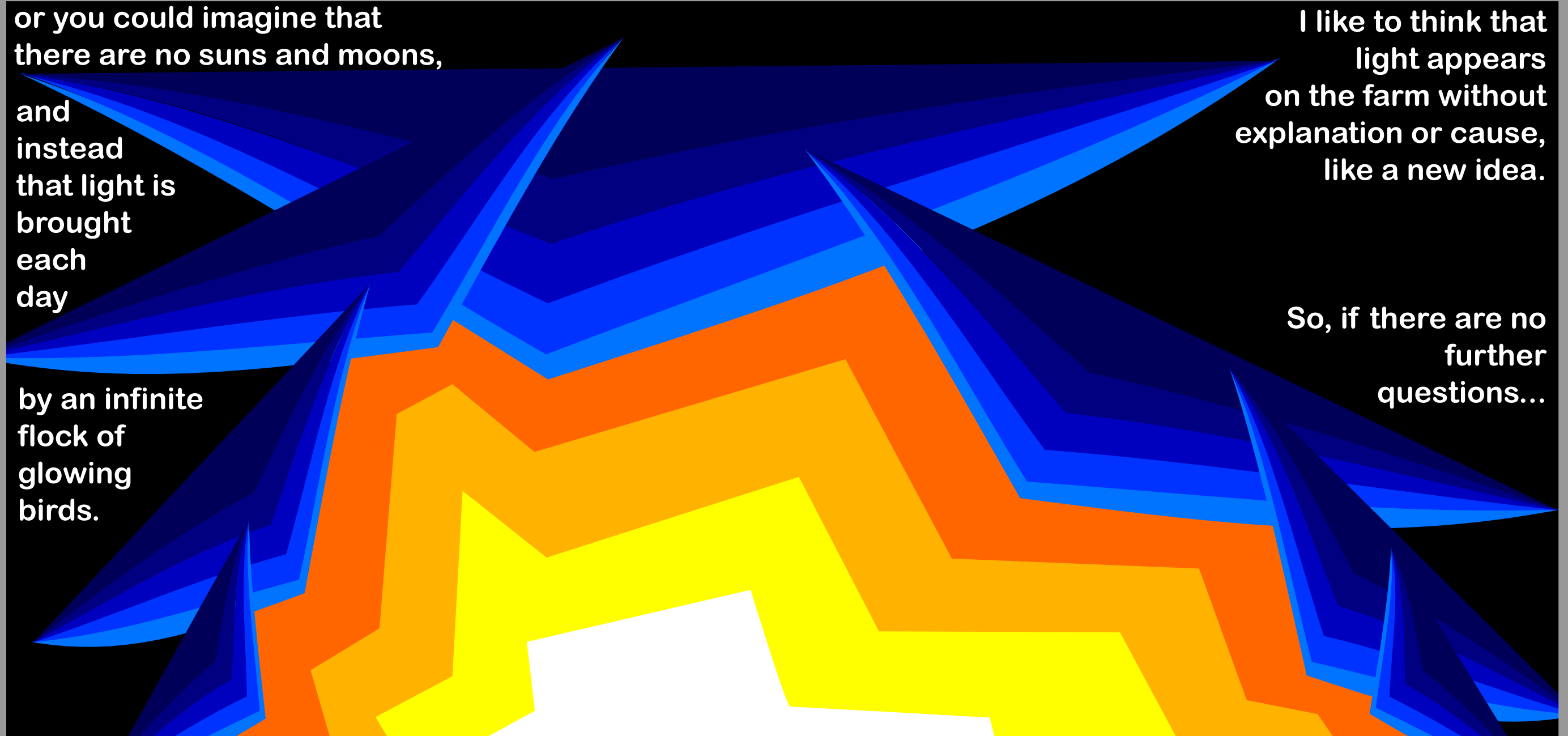
or you could imagine that  
there are no suns and moons,

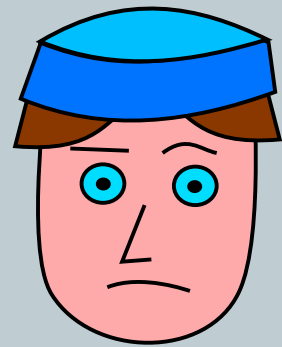
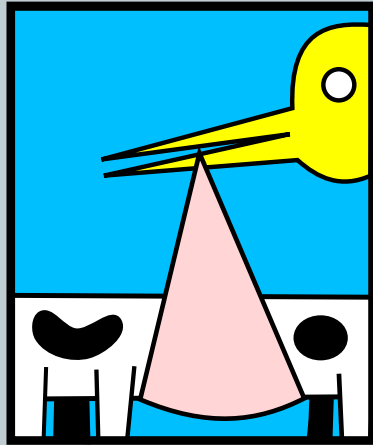
and  
instead  
that light is  
brought  
each  
day

by an infinite  
flock of  
glowing  
birds.

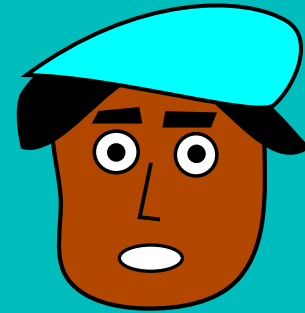
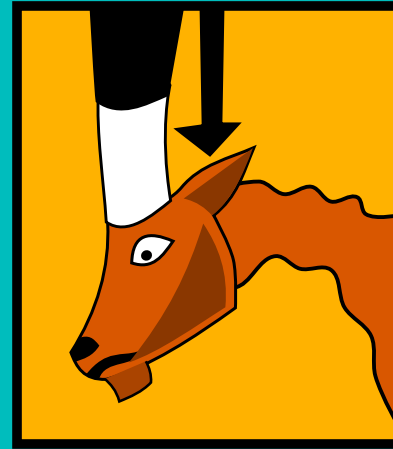
I like to think that  
light appears  
on the farm without  
explanation or cause,  
like a new idea.

So, if there are no  
further  
questions...

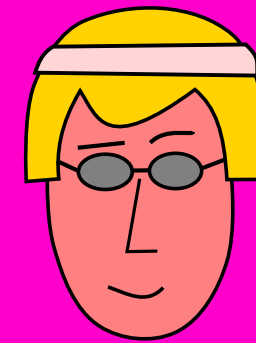
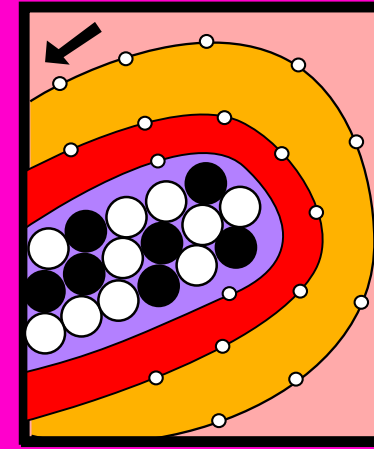




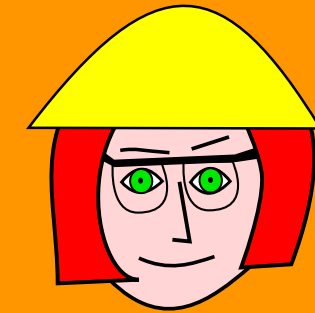
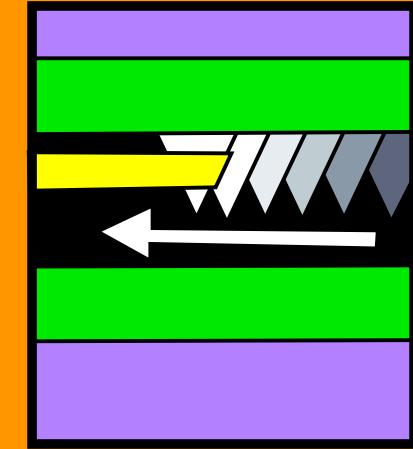
How did  
Gracie  
get on  
the farm?



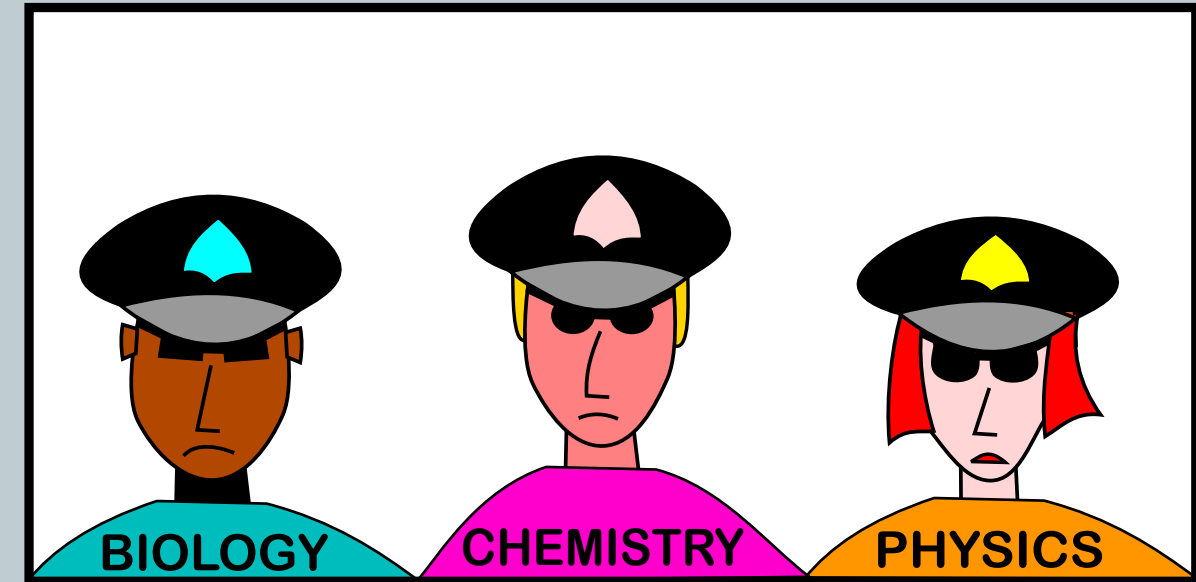
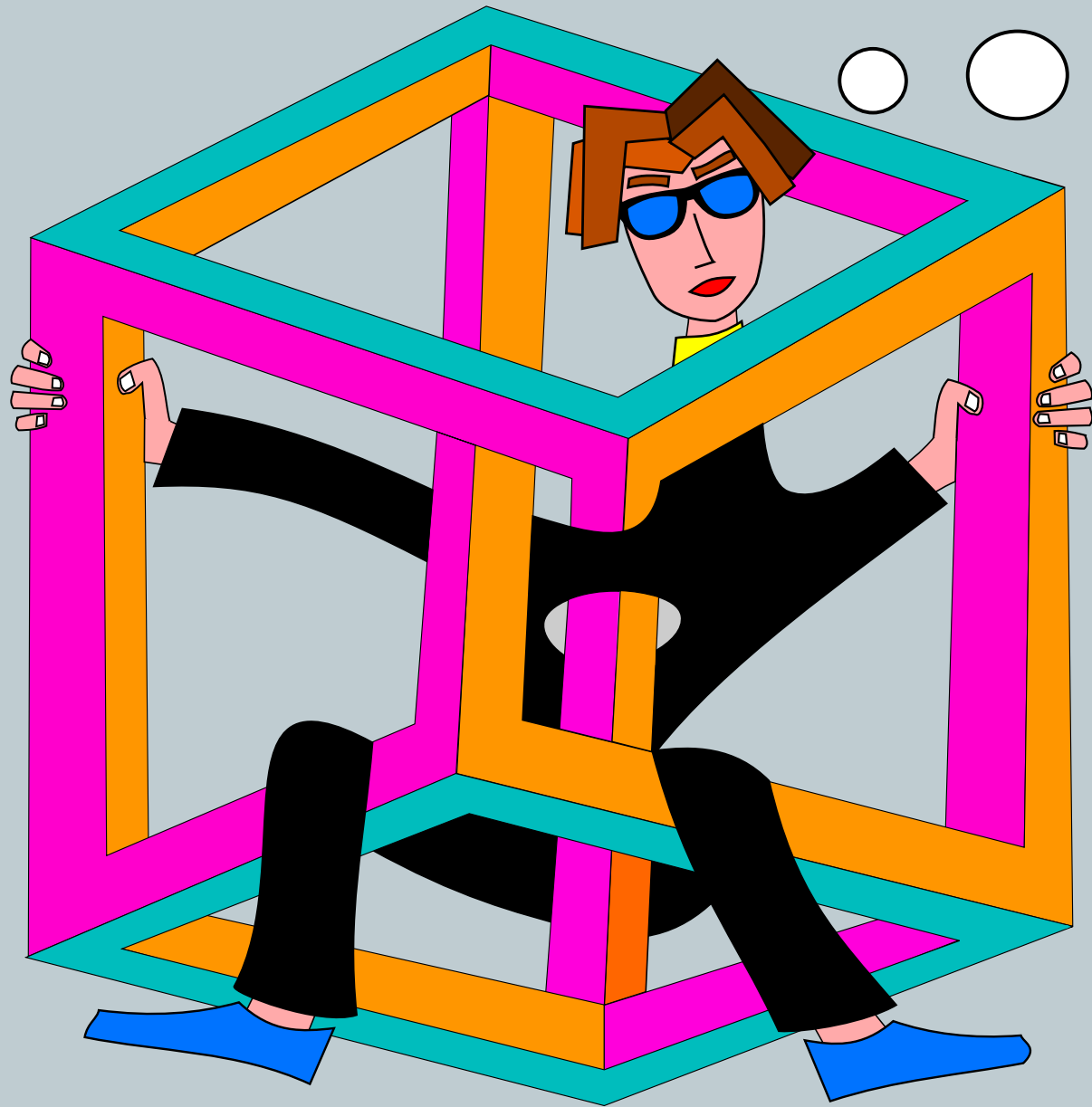
How can  
Ezekiel's  
neck support  
his infinite horn?



Would the farm  
have infinite  
atoms and/or  
infinite  
biomolecules?




Since most of his teeth would  
have to move faster than the  
speed of light, isn't  
Hammerwood's teeth-fixing  
process incompatible with  
the constraints of special  
relativity?



OK, OK, slow down! Let me try to answer all these questions at the same time. In our universe the sciences are relentless enforcers.

They seem to look into every situation, no matter how large or small, and make sure that it unfolds according to the law...



On the infinite farm,  
the sciences  
are more like  
negligent babysitters  
who check in from time  
to time but mostly  
just play around.

Meanwhile,  
out on  
the farm,...



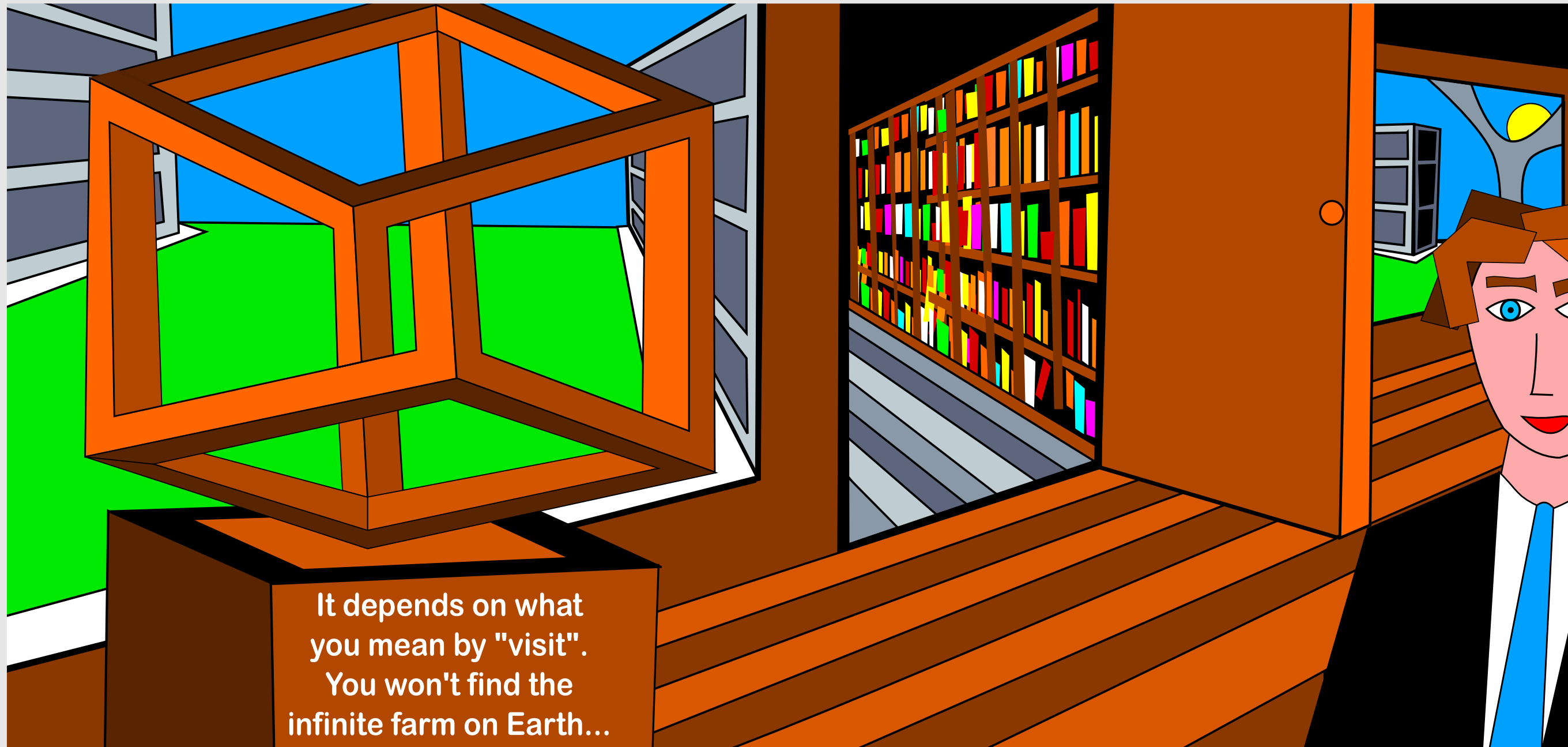
GEOMETRY  
makes  
the  
rules.

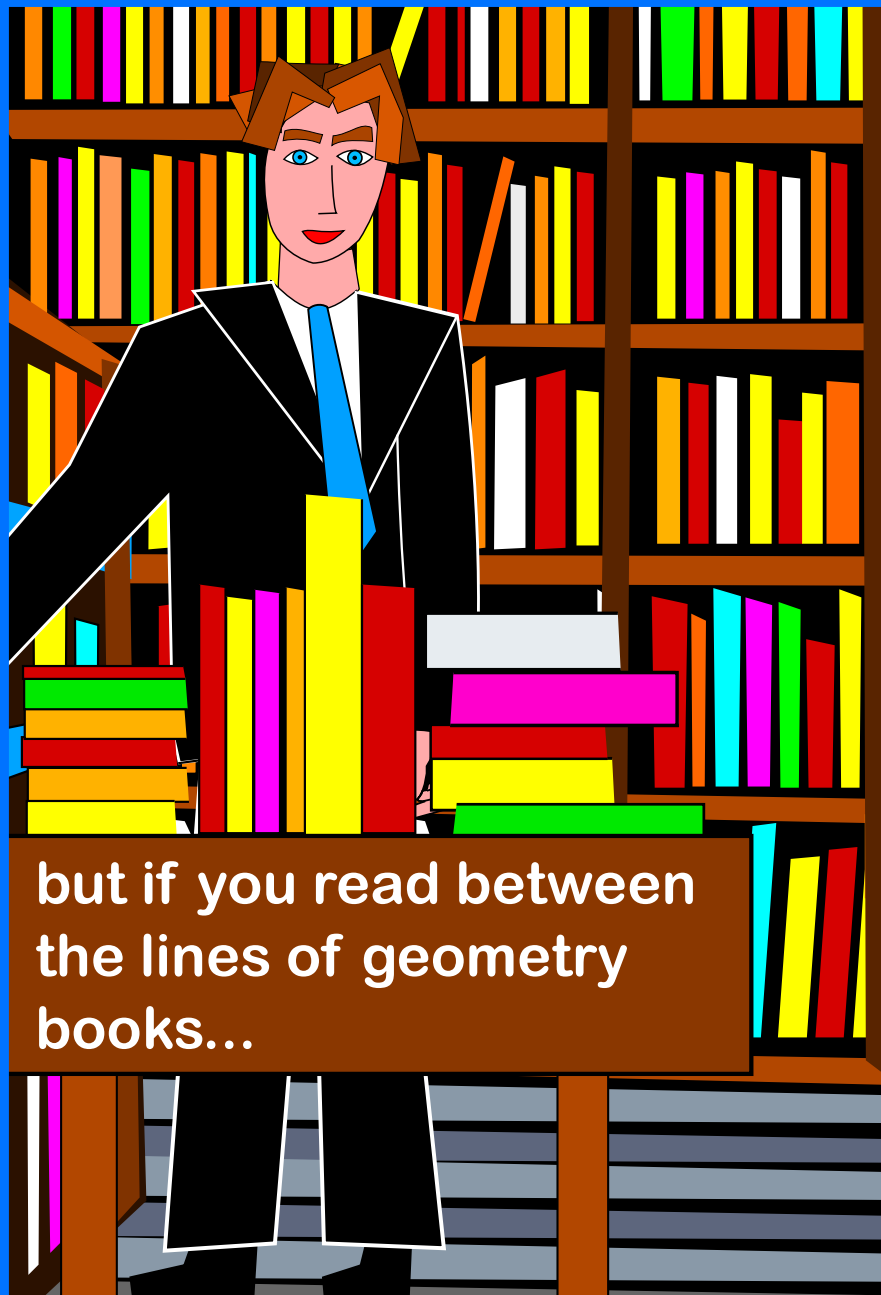
Well, I guess  
there's time for one  
last question.



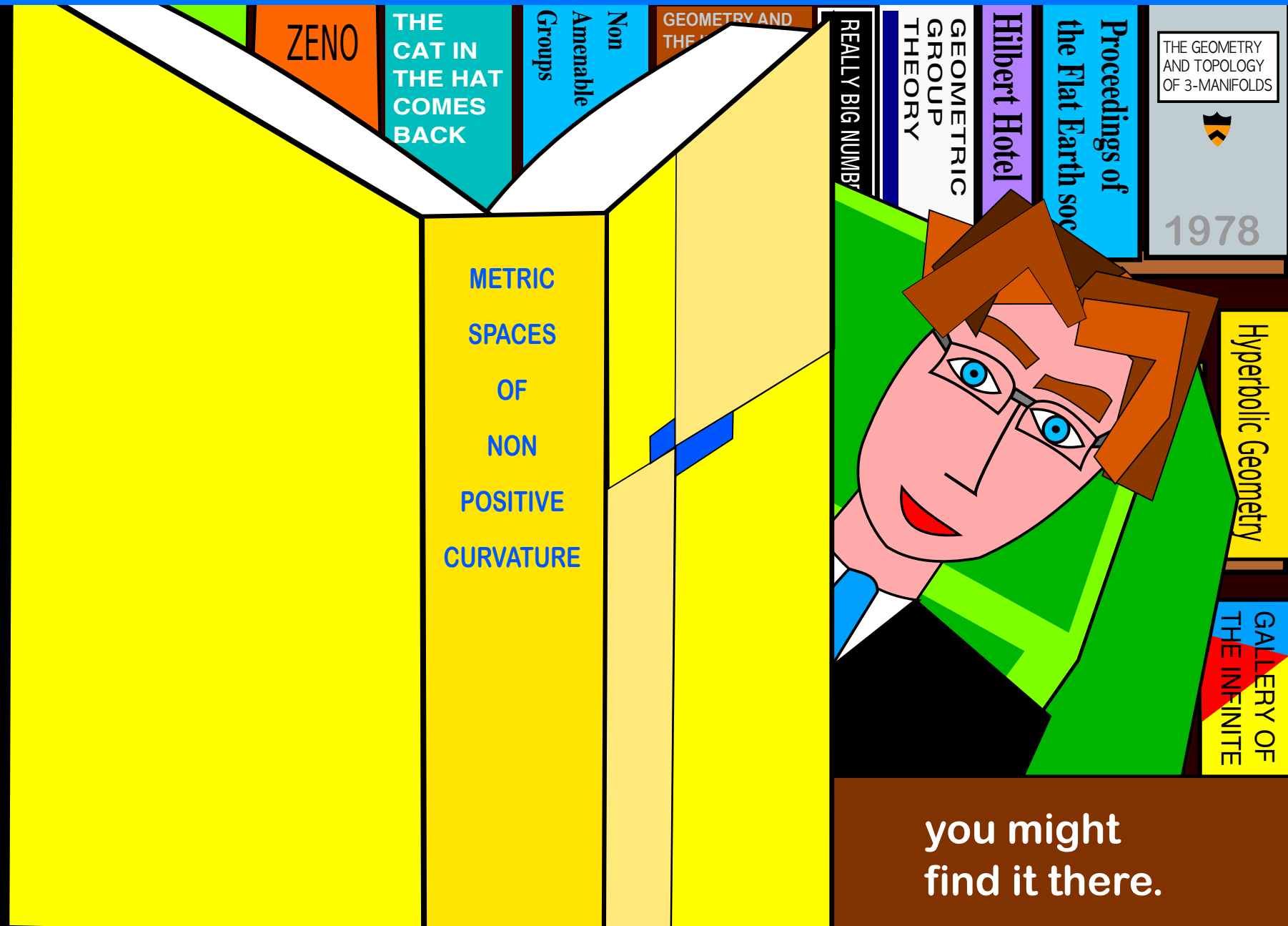
Can we VISIT  
the infinite farm?







but if you read between  
the lines of geometry  
books...



you might  
find it there.

About the author: I am the Chancellor's Professor of Mathematics at Brown University. In my spare time I like to draw, write computer programs, listen to music, walk on the beach, cycle, or go to the gym.

About the illustrations: I drew the pictures for this book using Inkscape, a great drawing program.

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