

Contemporary reviews of Flatland

We have made a few very minor editorial changes in these reviews including making (mathematical) word usage current and correcting obvious typos and misspellings. The footnotes are from the original articles; the endnotes contain our commentary.

1. *The Academy* (November 8, 1884), p. 302.
2. *The Athenaeum* No. 2977 (November 15, 1884), p. 622.
3. *The Athenaeum* No. 2978 (November 22, 1884), p. 660. (Abbott unmasked.)
4. *The Athenaeum* No. 2980 (December 6, 1884), p. 733. (Abbott's reply to No. 2977.)
5. *Nature* (November 27, 1884), pp. 76–77.
6. *The Spectator* (November 29, 1884), pp. 1583–1584.
7. *New York Times* (February 23, 1885), p. 3.
8. *The Literary News* (March 1885), p. 85.
9. *The Literary News* (April 1885), p. 123. (Abbott identified by name.)
10. *Science* (February 27, 1885), p. 184.
11. *Science* (April 3, 1885), pp. 265–266.
12. *The Literary World* (March 21, 1885), p. 93.
13. *Lippincott's Magazine* (May 1885), p. 528.
14. *City of London School Magazine* 8 (December 1885), 217–221.
15. *Nature* (February 12, 1920), p. 629. (Excerpt of Garnett's letter.)
16. *Saturday Review of Literature* (October 30, 1926), p. 254.
17. Reviews we'd like to have.

1. *The Academy* (November 8, 1884), p. 302.

Flatland is a world inhabited by beings whose experience of space is limited to two dimensions. In this book a native of this strange region has undertaken to describe its peculiarities to us dwellers in 'Spaceland.' It seems the male Flatlanders are plane rectilineal figures, varying in shape according to their position in the social scale, or, what in Flatland is the same thing, to their degree of intellectual development; the lowest class being isosceles triangles, and the highest class, or priesthood, being polygons which have so many sides that they are accounted circles. The Flatland women, being deplorably lacking in intellect, are not figures at all, but merely straight lines. Of course, the inhabitants of two-dimensional space cannot see each other as figures, but only as straight lines. For the means by which they can infer one another's true shape, and for the manners and institutions of Flatland, the reader must be referred to the book itself. The historian of Flatland is by rank and figure 'a square,' and he has a grandson, a clever hexagon, who one day startles him with

a suggestion that space may have a third dimension, and that beings may exist who are capable of seeing the inside of a closed figure. The notion is angrily rejected as absurd; but the ‘Square’ afterwards undergoes a miraculous experience that introduces him to the threefold space which he was previously unable to imagine. Guided by the analogy of his own experience, he ventures to suggest to the inhabitants of ‘Spaceland’ that a fourth dimension may have real existence, though it is to them as inconceivable as the third dimension once was to himself. The ‘Square’ has forgotten to tell us by what means he has managed to make himself intelligible to tri-dimensional mankind, and one or two other weak points might easily be found in his story; but, on the whole, the idea is very cleverly worked out, with many happy satiric touches, and the book is much more entertaining than this account of it will lead the reader to suppose.

2. *The Athenaeum* No. 2977 (November 15, 1884), p. 622

That whimsical book *Flatland* by a Square (Seeley & Co.), seems to have a purpose, but what that may be it is hard to discover. At first it read as if it were intended to teach young people the elementary principles of geometry. Next it seemed to have been written in support of the more transcendental branches of the same science. Lastly we fancied we could see indications that it was meant to enforce spiritualistic doctrines, with perhaps an admixture of covert satire on various social and political theories. The general purport of it is to show how being shaped like a square, born and bred in a world in which everything took place on a plane surface, and where consequently only two dimensions were conceived, obtained by a sort of revelation knowledge of a third dimension. He has previously in a dream studied the conditions of existence in a world of one dimension, where everything is a line or point, and nobody can pass any one else. There is some ingenuity in the way in which these conceptions are worked out, but it is rather spoilt to the mathematical mind by the conception (which, indeed, was unavoidable) of lines and points as objects which can be seen. Of course, if our friend the Square and his polygonal relations could see each other edgewise, they must have had some thickness, and need not, therefore, have been so distressed at the doctrine of a third dimension. There is something rather funny in the idea that a being of n dimensions, when addressed by a being of $n + 1$, fancies the voice which he hears to proceed from his own inside; but no doubt it is in strict harmony with facts, and probably represents what we should all feel if we got into a region where it was possible to tie a knot in a closed loop of string, as it is in the world of four dimensions. When we saw the feat performed we should doubtless be as much surprised as our Square was when the Sphere told him the contents of his house without opening the door or taking off the roof. If we came back and told about it, we should, equally without doubt, fare much as the unlucky narrator of this history did.

3. *The Athenaeum* No. 2978 (November 22, 1884), p. 660.

Literary Gossip. That curious little book *Flatland*, which we noticed last week is said to be the production of the head master of a well-known school.

4. *The Athenaeum* No. 2980 (December 6, 1884), p. 733.

THE METAPHYSICS OF FLATLAND.

Flatland, the State Prison, Nov. 28, 1884.

I write from a world that has been truly and literally described as “weary, stale, *flat*, and unprofitable,” — from the land of Two Dimensions, some of the characteristics of which I have recently endeavoured to describe in a little treatise entitled *Flatland*.

Into the dimness of my dull existence in this region there has penetrated a notice of my work which appeared in a recent number of the *Athenaeum*, and which raises a neat question — shall I say metaphysical or psychological? — which may possibly interest your readers.

Your not unfriendly, but, as I venture to think, to hasty critic, while complimenting me on the ‘ingenuity’ of my simple description of my native land, and while admitting that the incidents recorded in my history, though ‘funny,’ are nevertheless ‘strictly according to facts,’ has, nevertheless, cast an implied censure on my intelligence, and on that of my countrymen, by declaring that, though we think we are of Two Dimensions, we are really of Three, and ought to know it. The narrative is spoilt, he says, ‘for mathematical minds,’ because any *visible* line must really have thickness as well as length; and therefore all our so-called plane figures, besides having length and breadth, must really have some degree of thickness, or height — in other words a Third Dimension; and of this, he implies, we ought not to be ignorant.

I admit your critic’s facts, but I deny his conclusions. It is true, no doubt, that we really have a Third Dimension, just as it is also true that you have a Fourth. But just as you are not aware that you belong to the Fourth Dimension, so neither are we aware, nor can we be made logically aware, that we belong to the Third.

A moment’s reflection will make this obvious. Dimension implies measurement. Now, our lines are so thin that they cannot be measured. Measurement implies degrees, the more and the less; but all our lines are equally and infinitesimally thin, or thick, whichever you please to call it; so that we in Flatland can neither measure their thinness, nor even take cognizance of it. Where you speak of a line as being long and thick (or thin), we speak of it as being long and *bright*; ‘thickness’ (or ‘thinness’) never enters our heads, and we do not know what you mean by it. I knew what it meant once, during the few hours I spent in Spaceland; but I cannot realize it now. I take it on trust; but I cannot now make a mental image of it even to myself, much less to my countrymen.

Does this puzzle you? Then put yourself in my place. Suppose a being of the Fourth

Dimension, condescending to visit you, were to address you thus: “You creatures of Three Dimensions see a plane (which is of Two Dimensions) and you *infer* a solid (which is of Three); but in reality what you call a plane has another Dimension of a kind not known to you;” what would you reply? Would you not call for a policeman to see your visitor safely locked up in some asylum?

Well, precisely this has been my reception when I have attempted to demonstrate the facts insisted on by your critic. Only yesterday, when the Chief Circle (in other words the Chief Priest) paid his annual visit to my prison, I endeavoured to prove to him that the Figures which we saw around us had a Third non-recognized Dimension, being not only long and broad, but also what you in Spaceland call ‘high.’ What was his reply? Simply this: “Dimension implies measurement. You say I am ‘high’; measure my ‘high-ness’ and I will believe you.” I was crushed, and he left the room in triumph.

Sir, I am a humble Square, and I do not deny the superiority of your critic, who is doubtless a Cube; I impugn neither the exactness of his mathematics nor the regularity of his proportions; in the language of Spaceland, I am ready to admit that he is a regular Cube and no mistake. But I respectfully submit that his knowledge of human nature is not equal to his knowledge of mathematics. He has forgotten that we are all alike — Points, Lines, Squares, Cubes, Extra-Cubes, whether of no Dimensions or of many dimensions — liable to the prejudices of our several Dimensions, brothers in error; as one of your own poets also has said, “One touch of nature makes the world akin,” meaning thereby not one world only, but all worlds, and not excepting the favoured world of Three Dimensions. And I must say I take it ill that I should be, however gently, censured for appearing to be ignorant of a truth which I firmly apprehend by faith, and which I daily endeavour to inculcate upon others. —
A Square

*** If we understand the Square rightly, all that is wanting to make the Flatlanders realize a third dimension, and to settle circularism once for all, is a delicate micrometer. For he seems to admit that the edges of himself and his countrymen really are extended surfaces — as, indeed, appears from the fact which he elsewhere mentions, that they were capable of receiving colour. He is not, therefore, in the same position with regard to the third dimension as we of this world with regard to a fourth. The truth is, it may be suspected that our Square, having once in some measure grasped the conception of three-dimensioned space, cannot now wholly divest himself of it. He thinks, so to speak, in three dimensions. For instance, he talks in one place of hearing the sound of his wife’s retreating footsteps, a bold metaphor indeed to apply the motion of a line on a plane. But, with a degree of intellectual insincerity probably unconscious, certainly pardonable in a person situated as he is, he thinks it necessary to persist in saying that he apprehends by faith a truth which he has really learnt from the evidence of eyesight; thus making a serious confusion between the

functions of faith and sense. The Square does his reviewer too much honour in supposing him to be a regular cube. The best he can claim to be is a rectangular parallelepiped; and he finds it hard enough to live up to that configuration in space of the kind he knows, so that he is content to do without speculations as to the ways of beings in worlds of more or fewer dimensions.

5. *Nature* (November 27, 1884), pp. 76–77.

We live in an age of adventure. Men are ready to join in expeditions to the North Pole or to the interior of the African continent, yet we will venture to say that the work before us describes a vast plain as yet untrodden by any Fellow of the Royal Geographical Society, and teeming with a population of which no example has figured in any of our shows. A few years ago a distinguished mathematician published some speculations on the existence of a book-worm “cabin’d, cribb’d, confin’d” within the narrow limits of an ordinary sheet of paper, and another writer bewailed “the dreary infinities of homaloidal space.” A third remarks, “there is no logical impossibility in conceiving the existence of intelligent beings, living on and moving along the surface of any solid body, who are able to perceive nothing but what exists on this surface and insensible to all beyond it.”[1] How delighted Prof. Helmholtz will be to find, if this Flatland writer is worthy of credence, his conjecture is thus verified. ‘Flatland’ is not the real name of this unknown land (the secret is not divulged), but it is so called here to make its character clear to us Spacedenizens. It is a noteworthy fact that one at least of the Flatlanders expresses himself in remarkably correct English, and singularly after the manner of an ordinary Space-human being; and further, though — we regret to have to record it — as a martyr in the cause of the truth of a third dimension, he has spent seven long years in the State jail, yet these memoirs have in some mysterious manner found their way into our hands. There is hope then that some one of our readers may yet expatiate in the broad plain, though the penalty will be, we fear, that he must first become as flat as a pancake and then see to it that his configuration (as a triangle, square, or other figure) is regular. This latter is a *sine qua non* in Flatland, because, whatever you are, your configuration must be regular, or woe betide you, and you will shuffle off your mortal coil incontinently.

We will not stop to inquire how this and that have come about, but will endeavour to lay before our readers some of the features of this (to us) new world, though we are informed that it has just entered upon its third millennium.¹

In Flatland there is no sun nor any light to make shadows, but there is fog. This, which

¹ From the secret Archives it appears that at the commencement of each millennium a Sphere descended into the midst of the Council of Circles proclaiming the great truth for the attempted teaching of which our author is in bonds.

we on this earth consider to be an unmitigated nuisance, is recognized in that other world “as a blessing scarcely inferior to air itself, and as the Nurse of Arts and Parent of Sciences.” If there were no fog, all lines would be equally distinct, whereas under present circumstances, “by careful and constant experimental observation of comparative dimness and clearness, we are enabled to infer with great exactness the configuration of the object observed.” It is a necessity of Flatland life to know the north (for instance, it is a point of good breeding to give a lady the north side of the way); this is determined in the absence of any heavenly bodies by a novel (we speak as a Spacedenizen) law of Nature, *viz.* the constancy of an attraction to the south; however, in temperate regions the southward attraction is scarcely felt, but here again Nature comes to the Flatlander’s aid. If he is in an inhabited region, the fact that the houses (mostly regular pentagons; squares and triangles are only allowed in the case of powder-magazines, barracks, and such like, for sufficient reasons) have their roofs towards the north, so that the rain, which always comes from that quarter, may run off and not damage the houses, will help him to get his north point. If, however, he is out in the country far away from trees and houses, there is no help for him until a shower of rain comes. We must now give some description of the inhabitants. The women are all straight lines; the men are other regular figures (if there be hopeless irregularity, which the hospitals cannot cure, then the man is put to death). The lowest orders, policemen, soldiers, and the *canaille*, are isosceles triangles, their mental calibre being determined by the largeness of smallness of the vertical angle. It is possible for an isosceles triangle to be developed into an equilateral triangle, or the offspring after a few generations may be so developed; in this class are the respectable tradesmen. The professional men and gentlemen are Squares — our author is a lawyer — and Pentagons. The Circles (that is, approximations more or less closely to that figure) are the nobility.

Another law of Nature in Flatland is that “a male child shall have one more side than his father, so that each generation shall rise (as a rule) one step in the scale of development and nobility.” Our author, as appears by the drawing on the cover, has four pentagonal sons and two hexagonal grandsons. We do not clearly gather where the one eye (for Flatlanders appear to be monocular) is situated, and how locomotion is effected we are not told. It can hardly be by such means as were once suggested by Prof. Clerk Maxwell [2], for in Flatland you must go steadily forward or dire may be the disaster you will inflict upon your neighbour. There seems to be no lack of Board schools, and there is at least one university, that of Wentbridge (we had by force of habit written Cambridge), where instruction is given in mathematics. A knowledge of this branch of study is obligatory, for since a Flatlander’s eye can only move in his world-plane, all the objects, human and otherwise, even the circular priests, appear to be straight lines, and the figure-angles have to be, at any rate approximately, correctly guessed at sight.

Before we close our notice we must return to the description of womankind. The women we have said are straight lines, hence they are very formidable, for they are like needles, and what makes them more to be dreaded is that they have the power of making themselves practically invisible, hence a Flatland female is “a creature by no means to be trifled with.” There are, however, certain regulations in force which diminish the dangers resulting from having a woman about the house. There is an entrance for her on the eastern side of the house, by which she must enter “in a becoming and respectful manner”; she must also, when walking, keep up her peace-cry, under penalty of death, and if she has fits, is given to sneezing, or in any way is liable to any sudden movement, there is but one cure for such movements, and that is instant destruction. Though involuntary and sudden motions are thus summarily dealt with, yet if she is any public place she must keep up a gentle “back-motion,” and thus she is less liable to be invisible to her neighbours. Happily fashion exercises its potent sway in Flatland female society, as elsewhere, for we learn that “the rhythmical, and, if I may so say, well-modulated undulation of the back in our ladies of circular rank is envied and imitated by the wife of a common Equilateral, who can achieve nothing beyond a mere monotonous swing, like the ticking of a pendulum.” Owing to their unfortunate configuration they are inferior in all good qualities to the very lowest of the Isosceles, being entirely devoid of brain-power, and they have “neither reflection, judgment, nor forethought, and hardly any memory.” This is but a poor account, but we must bear in mind that it is an *ex parte* description by a Square who may at some time have suffered a disappointment at the hands of a lady. We shall be glad (though we can hardly expect such a result) — now that tidings have come from this little-known country — if some female will favour us with her view of the state of matters in Flatland. At birth a female is about an inch long, a tall adult woman reaches about a foot. The length of the sides of an adult male of every class may be said to be three feet or a little more.

The book consists of two parts — *This World*, i.e. of Flatland, in twelve sections, and *Other Worlds*, in ten sections. The whole is very cleverly worked out, and many passages descriptive of events in the past history of the country at times force upon one the thought that after all the book may have been compiled by a Spacedenizen, and that he is quietly laughing in his sleeve and saying, “de te Fabula narratur.” However this may be, Flatlander or Spacelander, there is a slip in the note on p. 64, and for “Flatland” should be read “Spaceland.”

We commend ‘A Square’s’ book to any of our readers who have a leisure hour from severer studies, and we believe when they have read it they will say “the tenth part of the humour has not been suggested”¹ R. Tucker [3]

¹ We may mention as specially humorous the chapters in which the Square is initiated into

6. *The Spectator* (November 29, 1884), pp. 1583–1584.

Strange are the tales of travelers, decisive the effect of experience upon previous speculations, and marvelously appropriate the morals brought home from outlandish quarters. Such are the reflections suggested by the attractive little book now before us. It tells of a region more unfamiliar than that of giants or pygmies, of anthropophagi, or men whose heads do grow beneath their shoulders. It throws a light on the question of the nature of space, which will be eagerly welcomed by seekers after a Fourth Dimension; and it proves that the institutions and failings of the race which inhabits the strangest countries bear a curiously perverted resemblance to those of our own.

Mathematicians have long speculated on the nature of space, and some have even questioned its universality. While rejoicing in the fact that their hands were free to move upwards as well as sideways, they have speculated on the possibility of worlds whose inhabitants should to be limited in their movements to the three prosaic directions of forwards, sideways, and upwards. They have lamented over the absence of the invaluable sense which would enable a man to get out of a closed cage without passing through the top, the sides, or the bottom, and to read all the contents of a shut book without touching its cover. They have hoped in a future state to enjoy an extended felicity in Space of Four Dimensions. They have dreaded the worse than flat-fish fate of being eternally confined to a limbo of two. These comfortable doctrines have long been a speculation and a pious opinion to the few, and a stumbling-block and foolishness to the uninitiated. They will receive a fresh impulse from the works of the adventurous traveler whose melancholy history is now before us. It is true that he cannot tell of a world of Four Dimensions. What he describes is almost as wonderful and more intelligible. He depicts life in which there is no possibility of sideways motion, a world where a man meeting another in the street cannot pass him except by jumping over his head, a world which would look to us only a large-sized map, and men who would appear moving anatomical diagrams. Such is his native world, but to that world he has not been confined. He tells of his mysterious and painful initiation into the larger world of Three Dimensions. He tells us, too, of his vision of the world whose universe was confined to a line, and of his contempt for the beings whose range appeared to him as limited as those of his own countrymen appear to us. He tells of the still more limited being, itself its own universe, whose conceptions were limited to the point which itself occupied. These unique experiences he dedicates to the inhabitants of space in general, in the laudable hope of “thereby contributing to the enlargement of the imagination and the possible development of that most rare and excellent gift of modesty among the superior races of solid humanity.”

For a full description of these new worlds, their optics and physics, their ethics and

some of the mysteries of tri-dimensional space by the spherical stranger.

politics, we must refer to the treatise of our author. Flatland is inhabited by a race whose shape is that of geometrical figures. The women are straight lines, the men vary from triangles in the lower orders to circles in the highest. Regularity of figure is the fundamental fact upon which the whole social life of Flatland rests, and is enforced upon the race by legislation as stern as that of Pharaoh. The other social problems, including the position of women, the shape of houses, the education of children, and the suppression of useless speculation, are dealt with in an equally drastic spirit. We will give an example, which will at the same time show the liberal spirit of the author:—

[The reviewer quotes the first third of Section 12]

But the chief interest will lie in the initiation of our author into the mystery of the Third Dimension. In the matter-of-fact account of the unintelligible entrance into Flatland, of the terrible Sphere from whom no secrets were hid, there is a weird suggestiveness of the possibilities that beset us, and of a catastrophe that might at any moment, for anything we know to the contrary, befall ourselves. The Sphere enters the room, where all doors are shut, on a disinterested mission of enlightenment, and the climax of the interview, as narrated by the Square, is as follows:—

[The reviewer quotes the first two-thirds of Section 17.]

Conviction, as might be expected, is only produced by hurling the unfortunate Square out of his plane; but for his feelings in the new world to which he is so unceremoniously introduced, and for the unfortunate results of his subsequent futile attempts to explain his experiences to his fellow-countrymen, we have no room.

The book has obviously been a source of much pleasure to the writer, and may be safely recommended to any mathematician fond of paradox. It is very pleasantly got-up in paper, print, and cover. Much of it will also be read with amusement, as satire, by those who do not appreciate its scientific bearing, or as pure nonsense by those who are not searching for satire. The chief fault we have to find is a want of proportion by which one or two rather heavy dissertations, such as that upon sight recognition, occupy an unnecessary and alarming amount of space. The assumption of the author is worked out with wonderful consistency, and his mathematics are thoroughly sound, though they are disfigured by two or three slips in the use of technical terms.

7. *New York Times* (February 23, 1885), p. 3.

This is a delirious book. A. Square, having lost his balance with overstudying geometry, statics, and kinetics, and having become stark mad about a line, a triangle, a pentagon, and a hexagon, has written a story about them. Take a penny and lay it on a table and leaning over it look down on it, and it is a circle. Look at it another way, and it becomes a line. Having then your penny, your circle, and your line, construct a geometrical romance, and

carry out the action in Flatland, Spaceland, and Lineland. A. Square is always asking the reader to imagine what he can't imagine; for instance, that a female in his country is a line. Imagine a line panted and her eye red with the hinder half green. Now, fancy a priest whose mouth is at M , and whose front semicircle (AMB) is consequently colored red, while his hinder semicircle is green. But the women in Flatland decline being painted, and there is a color revolt. "How I Vainly Tried to Explain the Nature of Flatland" is the title of one of the chapters of this story, and there is a terrible row between a Line and a Point, and an awful catastrophe happens when a sphere sits down on A. Square. It's a very puzzling book and a very distressing one, and to be enjoyed by about six, or at the outside seven, persons in the whole of the United States and Canada. A. Square has a brother, and that brother "has not yet grasped the nature of the third dimension, and frankly avows his disbelief in the existence of a sphere." May we remark that we love that brother? and if he had not existence in this geometrical romance we should go many miles to shake hands with him. Some little sense is apparent in an appeal for a better education for women, but beyond that all the rest of Flatland is incomprehensible.

8. *The Literary News* (March 1885), p. 85.

"A well-known English writer has published anonymously an amusing little book called *Flatland : a Romance of Many Dimensions*, by A. Square. It is an effective satire on social differences, and on the assumption of absolute knowledge. The characters are geometrical figures. The scenes include Pointland, the country of no dimension; Lineland, the place of one dimension; Flatland, where two dimensions are known, and a third is considered impossible; and Spaceland, where three dimensions are essential to existence, but where those who believe in a fourth are called insane. The geometrical inhabitants of all these regions have social ambition, immense conceit, positive views in regard to woman and education, and in each phase of life there is absolute disbelief in the possibility of anything broader or better. The book is full of light, good-natured mockery and absurd extravagances. The believers in a fourth dimension, which explains the phenomena of Spiritualism, may find some comfort in it, and may be able to grasp the idea of a fifth, sixth, and seventh dimension. All solid men and women who read the book will lay it down with a feeling of gratitude that they live in a land of three dimensions, and will cling to their, it may be narrow, skepticism in regard to a fourth." — *Boston Advertiser*

9. *The Literary News* (April 1885), p. 123.

LITERARY MISCELLANY

Flatland — published by Roberts Brothers — was written by the Rev. Dr. Edwin A. Abbott, headmaster of the City of London School.

10. *Science* (February 27, 1885), p. 184.

— The modern mathematician finds the space of three dimensions, in which our visible universe is contained, entirely too contracted for his conceptions, and is obliged to imagine a space of n dimensions in order that his fancy may find room to disport itself. But it is a new idea, on the part of the novelist, to make the conceptions of transcendental geometry the basis for an amusing story. *Flatland, a romance of many dimensions*, by A. Square (Boston, Roberts Brothers, 1885), is in substance a description of life as a geometer might imagine it to be in space of one, two, or n dimensions. Readers of ‘Alice behind the looking-glass’ will not fail to notice the resemblance of the present work to that singular play of fancy. Curiously enough, a ‘scientific romance’ on the fourth dimension is just now announced in England by C. H. Hinton.

11. *Science* (April 3, 1885), pp. 265–266.

Flatland, to which we referred a short time ago, besides giving the general reader an easy view of the road by which the mathematician enters the world of n dimensions, contains also a clever picture of the ludicrousness of various social theories now under discussion, when pushed to their legitimate consequences. The inhabitants of that country have the shape of various plane figures, — triangles, squares, pentagons, and polygons, — and the degree of their intelligence is in direct ratio to the number of their sides; so that ‘intellectuality’ becomes synonymous with ‘angularity,’ and the circle is a member of the priestly order, — the highest class of all. Beyond the soldiers and the lowest class of workmen, who are triangles with only two sides equal, — a figure so irregular that it can hardly be considered human, — it is a law of nature that each male child shall have on more side than his father.

Evolution is thus a perfectly regular and definite process; and a man’s remoteness from the flat apes, his ancestors, can be known by simply counting the number of his sides. Any slight irregularity in a figure is equivalent to a moral imperfection; and to train up a child in the path of virtue is to keep him straight in a literal sense. If he is born with any marked unevenness, he must be taken to one of the regular hospitals for the cure of that disease, or he is in danger of ending his days in the state prison. There is no way of knowing whether a particular delinquency calls for punishment or regard as a means of reform. The author, a square, confesses that he is at a loss what course to pursue when one of his own hexagonal grandsons pleads as an excuse for his disobedience that a sudden change in the temperature has caused an unequal shrinking in his perimeter, and that the blame ought to be laid, not on him, but on his configuration, which can only be strengthened by abundance of the choicest sweet-meats.

The women in Flatland are straight lines. As they have no angles, they have no intellect; and as they have nothing to say, and no constraint of wit, sense, or reason to prevent their

saying it, their conversation is a great bore. To such an extent has the system of female non-education or quietism been pushed, that they are no longer taught to read, nor to master arithmetic enough to count the angles of their husbands or children. The author fears that this policy has been carried so far as to react injuriously on the men, who are obliged to lead a bi-lingual or even a bi-mental existence. They must be able to speak not only the female language of emotion, but also the male language of science, in which ‘love’ becomes ‘the anticipation of benefits,’ ‘duty’ becomes ‘necessity’ or ‘fitness,’ and other words are correspondingly transmuted. In the presence of women, moreover, the language used implies the utmost deference for their sex; but behind their backs they are both regarded and spoken of as being little better than ‘mindless organisms.’ The strain of this dual existence, it is believed, has some tendency to enfeeble the male intellect, and on that ground alone the author appeals to the authorities to reconsider the regulations of female education.

12. *The Literary World* (March 21, 1885), p. 93.

This brilliant *jeu d’esprit*, published anonymously in England but attributed by the *Spectator* to the eminent Shakespearian scholar and educator, Rev. Edwin A. Abbott of London, has deservedly attracted much attention across the water. The mathematically-inclined will particularly enjoy its attempt to work out, as consistently as possible, the picture of a world having only the two dimensions of length and breadth. For such is Flatland, as the author, an enlightened and traveled Square, calls his native land, to make its nature clear to us. It is like “a vast sheet of paper on which straight *Lines* [the women], Triangles, Squares, Pentagons, Hexagons, and other figures move freely about, on or in the surface, but without the power of rising above or sinking below it.” With vast ingenuity and occasional shy satirical allusions to life in our Space-Land, ‘A Square’ describes the climate, the houses, the women, the methods of recognition by feeling and sight, the ancient practice of painting, the universal color belt, and the final suppression of the chromatic sedition. The Priests of Flatland are Circles; they have almost complete control of all affairs, and their doctrine is summed up in a single maxim, “Attend to your Configuration.” They teach men to honor their grandsons most of all, and that women should be left without education.

The author has a vision of Lineland where there is only one dimension; he endeavors to convince a native that there is another world, of two dimensions, but the labor is in vain, as the King of Lineland’s senses and consciousness are alike incapable of grasping the second dimension. This is but a dream however. Hard upon the dream comes the invasion of Mr. Square’s house by a stranger from Spaceland, a Sphere, who after long and unsuccessful efforts to beat the notion of thickness into Mr. Square’s head, at last carries him off bodily to Spaceland to convince him by actual experience. Returning home, after another vision, this time of Pointland, Square undertakes to teach the existence of Spaceland, the Gospel of

Three Dimensions. But he is soon arrested for heresy, by order of the grand Council, and condemned to perpetual imprisonment. Yet worse than prison is his present inability again to grasp, himself, “the exact shape of the once-seen, oft regretted Cube; and in my nightly visions the mysterious precept, ‘Upward, not Northward,’ haunts me like a soul-devouring Sphinx.”

With so much wit and grace of style is this clever satire on the limits of our knowledge wrought out that we shall not be surprised to see it take a permanent place in literature. Our friends, the Spiritists, will be sure to hail it as a strong argument in behalf of their much talked-of fourth dimension. But we should be inclined to say that this was not the author’s intention; his aim is more general to satirize the infallibility too common in Spaceland, which confounds the thinkable and the possible universes. The trouble with the alleged fourth dimension of Spiritism would seem to be that it annihilates the third dimension, instead of adding to it.

13. *Lippincott’s Magazine* (May 1885), p. 528.

There is undeniable wit in *Flatland*, but it is a mathematical fantasy, and the prosperity of its jests must lie in the ear of the professor of trigonometry or some other branch of the exact sciences. Conundrums are delightful when a clever answer rewards some stout cudgeling of the brains; but a book of continuous and unanswerable conundrums mocks and humiliates even the bravest. *Flatland*, however, is not shrouded in complete and painful mystery: there is an occasional lifting of the cloud, when we see triangles and parallelograms as men walking, and we are ready to declare it an amusing social satire, when all at once the fog which reigns in that region shuts down, and all meaning and human likeness vanishes in unrecognizable whim. People who understand *Flatland* will henceforth form a cultus by themselves, like the pioneer readers of Mr. Browning, and will look down on their fellowmen. Whether those who are barred out from this paradise will pine to get in remains to be seen. In this dim Flatland, where no sun shines and universal fog reigns, there are two ways of recognizing the various mathematical figures who make up the population. “Feeling,” being rudimentary, belongs to the uneducated, while “seeing,” being a purely intellectual process, is the accomplishment of the aristocratic and academical. Women being straight lines, like needles, all point, are more safely approached by academical pure reason and inference than by actual contact or feeling. The woman question is, in fact, as interesting in Flatland as elsewhere. It was long ago decided that, since women were deficient in reason but abundant in emotion, they must not be treated as rational beings: hence they receive no education, except enough rudimentary mathematics to enable them to define the angles of their husbands and children. The supreme male sex have thus been forced to adopt a language which females can understand. “With women,” the author remarks, “we speak of

love, duty, right, and other irrational and emotional conceptions which have no existence and the fiction of which has no object except to control feminine exuberances.” Among men love implies “the anticipation of benefits.” Women are always spoken to in terms of adoration, but are alluded to in masculine circles as “mindless organisms.” There is, as may be seen, plenty of point and humor in this, and the development of the story leads to a narrative which bristles with good things.

14. *The Critic* (April 18, 1885), p. 185.

A mathematician might not unnaturally anticipate a delicious bit of satire on taking up *Flatland: A Romance of many Dimensions*, by A. Square. (Boston: Roberts Bros.) But we are confident that, after reading it, ‘the dreary infinities of homoloidal space,’ complained of by Clifford [4], will appear to him infinitely drearier than ever. For the non-mathematical reader the satire is too nebulous to be effective. An intelligent lady reader tells us it is a satire on geometry, while an equally intelligent mathematician comes to the conclusion that it is a satire on society. Now, a satire whose object is a matter of dispute reminds us of the fable in which the monkey unrolled his panorama before the audience in a darkened chamber, having forgotten to light the candle. In one instance, however, a light flashed upon us — *viz.*, on reading that women, in Flatland, were straight lines, and therefore dangerous to meet ‘end on.’ It occurs to us that in order to justify the title of ‘A Romance’ (defined by Webster as a ‘a kind of novel’), the writer will have to persuade Mr. Hawthorne to invert his statement that the aim of the novel is to hold up to humanity ‘the illusion of a lofty reality,’ and to substitute therefore, ‘the reality of a lofty illusion.’

15. *City of London School Magazine* 8(December 1885), 217–221.

We have strong reasons for believing that the author of the above is not unknown to most of our readers: that this is not the first or the most philosophical production of his pen: and, what is more to the point, that the name of A Square will be found in the Mathematical Tripos list for the year 186- by any one who will consult the Cambridge Calendar for that purpose. [5]

The work is dedicated “To the Inhabitants of Space in general . . . by a Humble Native of Flatland, in the Hope that, even as he was initiated into the Mysteries of Three Dimensions, having been previously conversant with only Two, so the Citizens of that Celestial Region may aspire yet higher and higher to the Secrets of Four, Five, or even Six Dimensions, thereby contributing to the Enlargement of the Imagination, and the possible Development of that most rare and excellent Gift of Modesty among the superior races of Solid Humanity.” We hope the author will excuse our placing before a certain section of the “Inhabitants of Space” — that important section known as “Our Readers” — a short account of his experiences,

contributing thereby to their amusement, and, it is to be hoped, to their mental edification as well.

Part I. — *This* world (from the point of view of A SQUARE) enlarges upon the nature of Flatland. For the benefit of our younger readers, we will give a short explanation of Flatland (or the World of Two Dimensions). Imagine a vast sheet of paper — the thinner the better — and on it, or *in* it (which comes to the same thing if the paper is very thin) a number of figures drawn — straight lines, triangles, squares, pentagons, hexagons, and others — moving about freely in the surface of the paper, but without the power of rising above or sinking below it. “Then,” says the writer, “you will have a pretty correct notion of my country and countrymen. Alas, a few years ago, I should have said “my universe;” but now my mind has been opened to higher views of things.”

The inhabitants of Flatland are distinguished by the number of their sides. Women are straight lines; soldiers and common workmen are isosceles triangles with very short bases; the middle class consists of equilateral triangles; professional men and gentlemen are squares or pentagons; lastly come the several degrees of the nobility, beginning at hexagons, and rising by increase in the number of sides to unlimited polygons, which, we scarcely point out to students of Euclid, finally approximate to perfect circles, the order of priests.

As a general rule, a son has one more side than his father, but this does not apply to the lower classes. Nevertheless even they are not denied all hope of improving their condition. After a long period of military success or diligent labour, it is generally found that the more intelligent among the soldier and artisan classes manifest a slight increase of their base, and a shrinking of their equal sides, thus bringing them nearer to the equilateral type. “A wise ordinance of nature has decreed that, in proportion as the working classes increase in intelligence, knowledge, and all virtue, in that same proportion their acute angle (which makes them physically terrible) shall increase also, and approximate to the harmless angle of the equilateral . . . How admirable is this law of compensation!”

The inhabitants of Flatland have two methods of recognizing one another. Firstly, by the ordinary three-dimensional process of sight, considerably modified by the exigencies of two-dimensional physical laws. On the least reflection it will be patent to any one that an inhabitant of Flatland can only see another as a *line* — reduced to a *point* in the case of a woman coming directly towards him. Whence the law in well-regulated states that a woman shall keep herself perpetually oscillating to disclose her whereabouts to the other sex, and save them from sudden and invisible annihilation. The aids to estimation of distance afforded by differential muscular effort and parallax displacement appear to require a brain of three dimensions, and presumably an intelligence of more than three. Circles, squares, etc., are distinguished *through fog*, which is introduced much more opportunely in this universe than in ours, by the rapidity with which the line seen shades off at its extremities.

The second method is known as “feeling,” which consists in feeling the angles of the person to be recognized, and estimating the amount of their acuteness or obtuseness, as the case may be. This method is in favour with the lower classes, because the other requires a somewhat prolonged education. The following incident will serve to illustrate the principles described, as well as their danger:—

“I have heard that my excellent grandfather — one of the least irregular of his unhappy isosceles class, who indeed obtained, shortly before his decease, four out of seven votes from the Sanitary and Social Board for passing him into the class of the equal-sided — often deplored, with a tear in his venerable eye, a miscarriage which had occurred to his great-great-great-grandfather, a respectable working man with an angle or brain of $59^{\circ}30'$. According to his account, my unfortunate ancestor, being afflicted with rheumatism, and in the act of being felt by a polygon, by one sudden start accidentally transfixed the great man through the diagonal; and thereby, partly in consequence of his long imprisonment and degradation, and partly because of the moral shock which pervaded the whole of my ancestor’s relations, threw back our family a degree and a half in their ascent towards better things. The result was that in the next generation the family brain was registered at only 58° , and not till the lapse of five generations was the lost ground recovered, the full 60° attained, and the ascent from the isosceles finally achieved. And all this series of calamities from one little accident in the process of feeling.”

Space, to say nothing of a feeling that the author will not thank us for disclosing too much of its contents, forbids us to linger over Part I., with its account of the Universal Colour Bill, to aid recognition by sight, and the Suppression of the Chromatic Sedition that ensued. We must hurry on to the second, and, on the whole, more interesting part of the book, — *Other Worlds*.

On the night of the last day but one of the year — 1999, according to Flatland chronology, *our SQUARE* — having to follow his experiences for some little time we feel justified in calling him ours — had a dream, in which he saw Lineland, and, having accosted the monarch thereof as “woman,” for such he naturally appeared to be, discovering his mistake, endeavoured to enlarge the said monarch’s ideas, with very little result, especially since the monarch could not even see his informant unless the latter crossed the path of the line which constituted the former’s universe. The most striking feature of Lineland is the impossibility of passing your neighbours; nothing but oscillatory motion within small intervals is possible.

On the last night of 1999 *our SQUARE* was sitting alone with his wife, musing on the events of the past and the prospects of the coming millennium, but more particularly pondering over some words that had casually issued from the mouth of his youngest grandson, a most promising young hexagon, of mathematical attainments far above the average, who, when told that 3^2 represented the number of square inches in a square whose side was 3 inches

long, had perversely suggested that 3^3 must represent “a square of 3 inches each was, moving somehow parallel to itself (but I don’t see how) making a something else (but I don’t see what) of 3 inches every way” — a logical deduction which somewhat ruffled his grandfather. While in this state of mind he is visited by a sphere, who naturally appears to him as a circle, but possessed of the remarkable power of increasing his size from a point to a large circle, and *vice versâ*, and the still more remarkable power of vanishing at will. The stranger in vain seeks to impress him with the fact that there is a direction “up and down,” not to be confounded with north and south, and only enrages him by taking a tablet out of a locked closet [from *above*] and by giving him a dig in his stomach, or *area*, which he had hitherto naturally supposed inaccessible. At last the sphere loses all patience, and hurling him perforce out of his plane conducts him above it, and lets him look down on it and see the *inside* of everything, men and women not excepted. Then, following up his advantage by argument, the sphere makes his convert and replaces him in Flatland, coming to him again, however, and this time showing him Pointland, whose monarch can only ring the changes on one all-embracing sentiment, “It fills all space,” the *It* alluded to being of course himself. Our SQUARE is then left by the sphere with the watchword, “Upward, not Northward,” to preach the new “Gospel of the Third Dimension” amongst his fellow-countrymen, a proceeding which brings down upon him the strong arm of the law, to whose tender mercies he is consigned, and where we must leave him, the prey of a diseased imagination in the eyes of his countrymen, but in reality, as we know, a pitiable example of that pitiable class of individuals whom the world persistently misunderstands and rejects, and of whose genius it is not worthy.

We must again crave pardon of the author for extracting the pith from his romance, and would recommend any, especially mathematicians, who might appreciate a profitable diversion from the all-absorbing festivities of the Christmas season, to invest in the book. Its style in particular will amply repay them by its lucid simplicity and suppressed humour.

The moral of the story, if indeed it have one, must lie in the existence of space of more than three dimensions. But not even the concentrated wisdom of a president of the British Association has been able as yet to do more than suggest in the most provoking way that he himself has some conception of a fourth dimension.[6] Such an assertion is of course incontrovertible; but it does not help us much. The only process by which it might be possible to attain to any conception is that of analogy. Whether we regard four dimensions as the way from the inside to the outside of a closed surface without passing through it, or as space the section of which by three-dimension space is a limited solid, i.e. space whose *shadow* is of three dimensions; in both of these, and in similar arguments, we are confronted by a total lack of imagination to grasp the ideas and “body them forth,” an expression that has in itself a decided smack of three dimensions.

If, however, at any time the author is fortunate enough, in his capacity of a solid, to receive a revelation from the universe next above us in the continuous scale, we entreat him to lose no time in transferring to paper and transmitting to posterity his adventures in that region also. We at least shall be grateful to him.

15. *Nature*(February 12, 1920), p. 629.

The following is extracted from a ‘Letter to the Editor’ by William Garnett.[7] This excerpt was reproduced in Garnett’s introduction to the 1926 edition of *Flatland*.

Some thirty or more years ago a little *jeu d’esprit* was written by Dr. Edwin Abbott entitled *Flatland*. At the time of its publication it did not attract as much attention as it deserved. Dr. Abbott pictures intelligent beings whose whole experience is confined to a plane, or other space of two dimensions, who have no faculties by which they can become conscious of anything outside that space and no means of moving off the surface on which they live. He then asks the reader, who has consciousness of the third dimension, to imagine a sphere descending upon the plane of Flatland and passing through it. How will the inhabitants regard this phenomenon? They will not see the approaching sphere and will have no conception of its solidity. They will only be conscious of the circle in which it cuts their plane. This circle, at first a point, will gradually increase in diameter, driving the inhabitants of Flatland outwards from its circumference, and this will go on until half the sphere has passed through the plane, when the circle will gradually contract to a point and then vanish, leaving the Flatlanders in undisturbed possession of their country (supposing the wound in the plane to have healed). Their experience will be that of a circular obstacle gradually expanding or growing, and then contracting, and they will attribute to *growth in time* what the external observer in three dimensions assigns to motion in the third dimension. If there is motion of our three-dimensional space relative to the fourth dimension, all the changes we experience and assign to the flow of time will be due simply to this movement, the whole of the future as well as the past always existing in the fourth dimension.

16. *Saturday Review of Literature* (October 30, 1926), p. 254.

Dr. Abbott was out for fun when he wrote his friendly little geometrical romance, and it is good to see that the old wine is no worse for its new bottle. It is still a pleasant tonic, and an excellent stimulant for boys. Hitherto, only a few have enjoyed *Flatland*. It is now a pleasure in store for many.

Yet there is oddity in its reappearance at this time. The obvious reason for republishing is that in recent years we have waked up to the importance of what is loosely called “the” fourth dimension. An ingenious and easy narrative, introducing a fourth dimension by simple geometrical analogy, putting its eye-straining argument in words of one syllable, is therefore sure of a sale. *Tanquam ex ungue leonem*. I suspect Basil Blackwell of this cool logic. He

must be at the bottom of it. It is a shrewd notion, so far as publication is concerned. By all means let us buy the book, in this time of scientific quickening. But let us not be confused in reading it. The introduction suggests that Dr. Abbott was a prophet paving the way for the revelation of the theory of relativity; this is a gallant claim which ought to be denied. An A B C is given here, but so far as progressive scientific thought is concerned, it is an A B C of the wrong alphabet. The words of one syllable are in the wrong language. It is helpful, in that mental exercise is beneficial; but not more directly. One may go further, and say why. It is because *Flatland* is in the kingdom of literature, and not in the kingdom of science. The quality of thought behind the little book is not a quality of thought which is successful in scientific theory. *Flatland* has not been without influence; but its influence cannot be traced in such a book as Whitehead's *Science and the Modern World*. It can be traced in such a book as *Where the Blue Begins*.

An engaging fable, worthy of being remembered for its individual, literary merits — it thus appears somewhat oddly, among the books dealing with that rebuilding of scientific abstractions, which is the most notable architectonic achievement of our age. *Flatland* was invented as one would invent a game. It is the product of ingenuity, acting on material which has amusing possibilities. To paraphrase what Johnson said of Swift's *Lilliput*, "That rascal hasn't used an abstraction anywhere." *Flatland* is by no means up to *Lilliput*. The latter was an accident; the former is a straightforward *jeu d'esprit*, written in the age of ingenuity, in spirit very close to the early H. G. Wells or Jules Verne, and more loosely akin to some adventurers in mysticism. It is not in the stream of serious thought; but those who like backwaters will enjoy it. — Frank V. Morley [8]

17. Reviews we'd like to have.

- a. *Cambridge Review* **6**(1884/1885), lxxi?
- b. *American* **9:237**(February 21, 1885), p. 312
- c. *New Statesman*(July 10, 1926), p. 364
- d. *Boston Transcript*(October 30, 1926), p. 5
- e. *Outlook*(December 15, 1926), p. 508
- f. The following item [*Knowledge*(March 1, 1887), 116.] leads us to conjecture that *Knowledge* printed a review which Abbott found objectionable.

"A Square" sends a "wail from Flatland," or, in other words, the well-known Tridimensional, whose romances of Flatland have touched many students of quadri-dimensional mathematics, has sent a letter describing the anguish of "A Square" imprisoned in such cells as Mr. Garbett failed to escape from. We would publish this piteous lamentation were more of our readers quadri-dimensional; but the number of such readers is so limited, that we must ask the ingenious author to forgive us if we

refrain. Will he kindly regard his letter as printed in planes persistently perpendicular to those of the pages of *Knowledge*?

g. Reviews from which the following excerpts were taken.

[Dust jacket of the 1937 Little Brown edition of *Flatland*]

Some Diverting Contrasts of Opinion Concerning

FLATLAND

BRICKBATS

“Life is too short for it to be worth anybody’s while to attempt to solve or comprehend the solemn jokes, the labored jocoseness, which the unannounced author of this nondescript production perpetrates.”

“Perhaps the essay is the desperate face-tiousness of a long-suffering student of geometry.”

“... the thing is arrant and witless and inexcusable nonsense.”

“... it is mortally tedious ... we prescribe it as a soporific.”

“We frankly say that we are at a loss as to the meaning and purpose of this queer little book.”

“For the most part the reader will be puzzled to know what it is all about, and so prolix and dry is the style that he will hardly care to bother himself with guessing. It is a book of 155 pages. All that there is of any value in it might have been put into 10 pages.”

“Nothing is much more contemptible in this mundane sphere than to pretend to do, or to be, or to know when one can neither do, nor be, nor know, but contrariwise only make pretense, and is in a word a Humbug. Now, any man that shall pretend to know what this book means is of necessity a humbug.”

BOUQUETS

“With so much wit and grace of style is this clever satire on the limits of our knowledge wrought out that we shall not be surprised to see it take a permanent place in literature.”

“‘Flatland’ is one of the most clever and fascinating books of the season.”

“A work that will find a place among the cleverest satires of history.”

“A remarkable book which serves notice to man to keep away from Flatland. It is an original and great power.”

“The idea is well worked up and the little book is worthy of a place beside the *Lilliput*, *Brobdingnag* and the *Houyhnhnms* of the great dean of St. Patricks.”

“The book is mathematically correct, which adds to its remarkable cleverness.”

“Few could read this brochure without having the mind broadened in consequence of its perusal.”

“It is exceedingly clever and entertaining.”

“The satire is as intense as that of ‘*Gulliver’s Travels*’.”

Endnotes.

1] The ‘distinguished mathematician’ is Gauss, who in *Treatise on the Geometry of Curved Surfaces* (1827) encouraged his readers to imagine the sorts of measurements that would be made by intelligent flatworms moving along the surface of a membrane in space. The phrase ‘cabind, cribbd, confind’ is from *Macbeth*, Act III, scene 4:

Then comes my fit again. I had else been perfect,
Whole as the marble, founded as the rock,
As broad and general as the casing air;
But now I am cabind, cribbd, confind, bound in
To saucy doubts and fears.

In March 1873, W. K. Clifford closed his lecture, “The Postulates of the Science of Space,” with the sentence: “In fact, I do not mind confessing that I personally have often found relief from the dreary infinities of homaloidal space in the consoling hope that, after all this other may be the true state of things. (“This other” refers to a three-dimensional structure curved in four dimensions.)

Maxwell incorporated homaloidal space into the first stanza his poem *A Paradoxical Ode*:

My soul is an entangled knot,
Upon a liquid vortex wrought
By Intellect in the Unseen residing,
And thine doth like a convict sit,
With marlinespike untwisting it,
Only to find its knottiness abiding;
Since all the tools for its untying
In four-dimensional space are lying,
Wherein they fancy intersperses
Long avenues of universes,
While Klein and Clifford fill the void
With one finite, unbounded homaloid,
And think the Infinite is now at last destroyed.

In a lecture translated as *On the Origin and Significance of Geometrical Axioms*, Hermann von Helmholtz considered reasoning, two-dimensional beings which live and move on the surface of a solid body, trying to understand the intrinsic geometry of their world without benefit of a three-dimensional perspective.

2] We have not found this reference.

3] The only thing we know about Robert Tucker is that at the request of Clifford's widow, he edited *Mathematical Papers*, a compilation of Clifford's work.

4] Clifford, "The Postulates of the Science of Space."

5] The Tripos is the final honors examination for the B. A. degree at Cambridge University. In 1861, Abbott finished first in the Classical Tripos and was seventh 'Senior Optime' in the Mathematical Tripos.

6] The "president of the British Association" is J. J. Sylvester (1814–1897); the remark refers to Sylvester's address to the mathematical and physical section of the British Association at Exeter in August, 1869. An abridgement of this talk (including his reference to the fourth dimension) was published as, "A plea for the mathematician," *Nature* 1(December 30, 1869), pp. 237–239. The complete address with annotations is included in Sylvester's *The Laws of Verse*, pp. 101–152. In *Mathematical Visions*, pp. 55–56, Joan Richards echoed the reviewer's characterization of Sylvester's endorsement of the fourth dimension.

Below is an excerpt of Sylvester's address containing the "provoking suggestion that he himself has some conception of a fourth dimension."

I may mention that Baron Sartorius von Waltershausen (a member of this Association) in his biography of Gauss ('Gauss zu Gedachtniss'), published shortly after his death, relates that this great man used to say that he had laid aside several questions which he had treated analytically, and hoped to apply to them geometrical methods in a future state of existence, when his conceptions of space should have become amplified and extended; for as we can conceive beings (like infinitely attenuated bookworms* in an infinitely thin sheet of paper) which possess only the notion of space of two dimensions, so we may imagine beings capable of realising space of four or a greater number of dimensions.** Our Cayley, the central luminary, the Darwin of the English school of mathematicians, started and elaborated at an early age, and with happy consequences, the same bold hypothesis.

* I have read or been told (I believe erroneously) that eye of observer has never lighted on these depredators, living or dead. Nature has gifted me with eyes of exceptional microscopic power, and I can speak with some assurance of having repeatedly seen the creature wriggling on the learned page. On approaching it with breath or finger-nail it stiffens out into the semblance of a streak of dirt, and so eludes detection.

** It is well known to those who have gone into these views that the laws of motion accepted as a fact suffice to prove in a general way that the space we live

in is a flat or level space (a 'homaloid'), our existence therein being assimilable to the life of the bookworm in a flat page; but what if the page should be undergoing a process of gradual bending into a curved form? Mr. W. K. Clifford has indulged in some remarkable speculations as to the possibility of our being able to infer, from certain unexplained phenomena of light and magnetism, the fact of our level space of three dimensions being in the act of undergoing in space of four dimensions (space as inconceivable to us as our space to the supposititious bookworm) a distortion analogous to the rumpling of the page. I know there are many, who, like my honoured and deeply lamented friend the late eminent Prof. Donkin, regard the alleged notion of generalised space as only a disguised form of algebraical formulisation; but the same might be said with equal truth of our notion of infinity in algebra, or of impossible lines, or lines making a zero angle in geometry, the utility of dealing with which as positive substantiated notions no one will be found to dispute. Dr. Salmon, in his extension of Chasles' theory of characteristics to surfaces, Mr. Clifford, in a question of probability (published in the *Educational Times*), and myself in my theory of partitions, and also in my paper on Barycentric Projection in the *Philosophical Magazine*, have all felt and given evidence of the practical utility of handling space of four dimensions, as if it were conceivable space. Moreover, it should be borne in mind that every perspective representation of figured space of four dimensions is a figure in real space, and that the properties of figures admit of being studied to a great extent, if not completely, in their perspective representations. In philosophy, as in aesthetic, the highest knowledge comes by faith. I know (from personal experience of the fact) that Mr. Linnell or Madame Bodichon can distinguish purple tints in clouds where my untutored eye and unpurged vision can perceive only confused grey. If an Aristotle, or Descartes, or Kant, assures me that he recognises God in the conscience, I accuse my own blindness if I fail to see with him. If Gauss, Cayley, Riemann, Schaffli, Salmon, Clifford, Kronecker, have an inner assurance of the reality of transcendental space, I strive to bring my faculties of mental vision into accordance with theirs. The positive evidence in such cases is more worthy than the negative, and actuality is not cancelled or balanced by privation, as matter plus space is none the less matter. I acknowledge two separate sources of authority — the collective sense of mankind, and the illumination of privileged intellects. As a parallel case, I would ask whether it is by demonstrative processes that the doctrine of limits and of infinitely greats and smalls, has found its way to the ready acceptance of the multitude; or whether, after deducting whatever may be due to modified hereditary cerebral organisation, it is not a consequence rather of the insensible molding of the ideas under the influence of language which has become permeated with the notions originating in the minds

of a few great thinkers? I am assured that Germans, even of the non-literary class, such as ladies of fashion and novel readers, are often appalled by the hebetude of their English friends in muddling up together, as if they were nearly or quite the same thing, the reason and the understanding, in doing into English the words *Vernunft* and *Verstand*, thereby confounding distinctions now become familiar (such is the force of language) to the very milkmaids of Fatherland.

As a public teacher of mere striplings, I am often amazed by the facility and absence of resistance with which the principles of the infinitesimal calculus are accepted and assimilated by the present race of learners. When I was young, a boy of sixteen or seventeen who knew his infinitesimal calculus would have been almost pointed at in the streets as a prodigy, like Dante, as a man who had seen hell. Now-a-days, our Woolwich cadets at the same age, talk with glee of tangents and asymptotes and points of contrary flexure and discuss questions of double maxima and minima, or ballistic pendulums, or motion in a resisting medium, under the familiar and ignoble name of sums.

7] William Garnett was first boy in mathematics at the City of London School in the same year that H. H. Asquith, the future prime minister, was first boy in classics. Garnett went to Trinity College in Cambridge and became Demonstrator of Physics for the Cavendish Laboratory under Maxwell. Garnett had impressed Maxwell in his Tripos answers the previous year, when he was Fourth Wrangler. He collaborated with Lewis Campbell on *The Life of James Clerk Maxwell*(1882); that book shows Maxwell's continued interest in higher dimensions.

8] Frank V. Morley (1899–1980) was a publisher, editor, and writer. He was a co-founder of the British publishing company, Faber and Faber and was a vice-president and editor at Harcourt, Brace & Co. He wrote several books including *Literary Britain*; his brother, Christopher Morley, wrote the book, *Where the blue begins*. Morley's father (Frank Morley) was chairman of the mathematics department at Johns Hopkins University and President of the American Mathematical Society (1919–1920). He is remembered for one of the loveliest results in all of elementary geometry, now known as Morley's Theorem: The three points of intersection of the adjacent trisectors of the angles of any triangle are the vertices of an equilateral triangle.