Math 180 - First Assignment (Fall 2021)

Future assignments in this course will consist of problems from the course textbook, but the problems for the first week are provided below.

The **Self-Check Problems** are intended to help you practice and confirm that you're solving the problems correctly. Answers can be found at the end of this document (and for future assignments, in the back of the textbook). You do not need to submit solutions to Self-Check Problems, but you are still expected to solve them; the Collected Problems alone will not give you enough practice to reliably learn the course material.

Solutions to the **Collected Problems** should be written up clearly, showing your work, and handed in to your TA at the beginning of the first recitation session, on Tuesday, September 14. This work will count toward your homework grade.

SELF-CHECK PROBLEMS

For Problems 1 and 2, determine the derivative of the given function.

1. $f(t) = t \ln\left(e^t + 1\right)$

2. $g(x) = \sin\left(\sqrt{x^6 + 9}\right)$

For Problems 3-5, evaluate the given indefinite or definite integral.

3. $\displaystyle\int \frac{\sin 3\theta}{\sqrt{\cos 3\theta + 4}}\, d\theta$

4. $\displaystyle\int_5^6 \frac{1}{x^2 - 16}\, dx$

5. $\displaystyle\int_5^6 \frac{1}{\sqrt{x^2 - 16}}\, dx$

6. (a) Determine an equation for the sphere centered at $(7, -3, 2)$ with radius 4.
   (b) Is the point $(4, 0, 4)$ inside or outside this sphere? Explain how you know.

7. Let $\vec{v} = \langle -2, 1, 1\rangle$ and $\vec{w} = \langle 3, 4, 5\rangle$. Evaluate each of the following expressions.
   (a) $\vec{v} \cdot \vec{v}$
   (b) $|\vec{v}|$
   (c) $-3\vec{w}$
   (d) $\vec{v} \cdot \vec{w}$
   (e) $(6\vec{v} + \vec{w}) \cdot (6\vec{v} - \vec{w})$

8. A bird is attempting to fly northeast at a constant speed, but a wind blowing southward at 5 miles per hour blows the bird off course. If the bird's overall movement (incorporating its intended movement and the movement due to wind) is at a speed of $\sqrt{53}$ miles per hour, how fast would it have been traveling if there was no wind?

9. Consider the following four vectors:

   - $3\vec{\imath} + \vec{\jmath} + 8\vec{k}$
   - $\vec{\imath} + 2\vec{\jmath} + 8\vec{k}$
   - $2\vec{\imath} + 4\vec{\jmath} - \vec{k}$
   - $4\vec{\imath} + 2\vec{\jmath} - \vec{k}$

   (a) Two of these four vectors are orthogonal. Determine which two.

   (b) Determine the angle between the other two vectors. Use a calculator to round your answer to the nearest tenth of a degree.

10. Determine the vector projection of the vector $\langle 8, -3, 4\rangle$ onto each of the vectors $\vec{\imath}$, $\vec{\jmath}$, and $\vec{k}$.

11. An object weighs 10 pounds, so the force of gravity on the object is represented by the downward-pointing vector $\langle 0, 0, -10\rangle$. However, the object is on a slanted surface, so that the vector $\langle 1, -2, -5\rangle$ points straight into the surface.

    (a) Determine the magnitude of the portion of the gravity vector that points straight into the surface. (In physics, this quantity would be used to calculate friction.)

    (b) Determine the magnitude of the portion of the gravity vector that is orthogonal to your answer from part (a).

COLLECTED PROBLEMS (Turn these in!)

For Problems 1 and 2, determine the derivative of the given function.

1. $f(x) = e^{3x}\cos 4x$

2. $g(t) = \dfrac{3t+2}{t^2+4t}$

For Problems 3-5, evaluate the given indefinite or definite integral.

3. $\displaystyle\int \frac{3t+2}{t^2+4t}\,dt$

4. $\displaystyle\int xe^{5x}\,dx$

5. $\displaystyle\int_0^1 \frac{t^4}{(2t^5+3)^3}\,dt$

6. The equation $x^2+y^2+z^2+6x-4y-10z+34=0$ defines a sphere.
   (a) Determine the center and radius of this sphere.
   (b) Determine the two points on the sphere that have the highest and lowest z-coordinate.

7. Let $\vec{v} = \langle 2,-3,4\rangle$ and $\vec{w} = \langle 5,1,1\rangle$. Evaluate each of the following expressions.
   (a) $\vec{v}\cdot\vec{w}$
   (b) $|\vec{v}|$
   (c) $3\vec{v}-4\vec{w}$
   (d) $(10\vec{v})\cdot(20\vec{w})$

8. Consider the vectors $\langle c,3,4\rangle$ and $\langle 6,-c,2\rangle$, where c is an unknown constant.
   (a) Determine the cosine of the angle between these vectors if $c=0$. (You do not have to solve for the angle itself.)
   (b) Determine the value of c that would cause these vectors to be orthogonal.

9. Suppose that an object hangs from three ropes which point in the directions of the vectors $\langle 2,-1,1\rangle$, $\langle 1,3,1\rangle$, and $\langle -1,-1,1\rangle$. (Assume that the positive z-axis points directly upward.) If the tension in the first of these three ropes is 40 pounds, determine the weight of the object.

10. Let $\vec{v} = 3\vec{\imath}+\vec{\jmath}-2\vec{k}$ and $\vec{w} = 4\vec{\imath}-2\vec{\jmath}+3\vec{k}$.
    (a) Determine the vector projection of $\vec{v}$ onto $\vec{w}$.
    (b) Determine the vector projection of $\vec{w}$ onto $\vec{v}$.
    (c) Determine the vector projection of $\vec{v}$ onto $\vec{v}$. (This is not a typo.)

11. Let $\vec{u} = \langle 4,7\rangle$, $\vec{v} = \langle 2,7\rangle$, and $\vec{w} = \langle 3,5\rangle$.
    (a) Write $\vec{u}$ as the sum of a vector parallel to $\vec{v}$ and a vector parallel to $\vec{w}$.
    (b) Write $\vec{u}$ as the sum of a vector parallel to $\vec{w}$ and a vector orthogonal to $\vec{w}$.

ANSWERS TO SELF-CHECK PROBLEMS

1. $$\ln(e^x + 1) + \frac{xe^x}{e^x + 1}$$

2. $$\frac{3x^5 \cos(\sqrt{x^6 + 9})}{\sqrt{x^6 + 9}}$$

3. $$-\frac{2}{3}\sqrt{\cos 3\theta + 4} + C$$

4. $$\frac{1}{8} \ln\left(\frac{9}{5}\right)$$

5. $$\ln\left(\frac{3 + \sqrt{5}}{4}\right)$$

6. (a) $x^2 - 14x + 49 + y^2 + 6y + 9 + z^2 - 4z + 4 = 16$
   (b) Outside, because its distance from $(7, -3, 2)$ is $\sqrt{22} > 4$.

7. (a) 6
   (b) $\sqrt{6}$
   (c) $\langle -9, -12, -15 \rangle$
   (d) 3
   (e) 166

8. $7\sqrt{2}$ miles per hour

9. (a) $\vec{\imath} + 2\vec{\jmath} + 8\vec{k}$ and $4\vec{\imath} + 2\vec{\jmath} - \vec{k}$
   (b) $\arccos\left(\frac{2}{\sqrt{1554}}\right) \approx 87.1$ degrees

10. $8\vec{\imath}$, $-3\vec{\jmath}$, and $4\vec{k}$

11. (a) $5\sqrt{30}/3$
    (b) $5\sqrt{6}/3$