DRP Project Descriptions Spring 2020

1. *An introduction to Teichmüller spaces* by Y. Imayoshi & M. Taniguchi

   The goal of the project is to learn what Teichmüller spaces are and what their theory is all about. Topology and complex analysis feature heavily in this theory, so the project will be ideal for students who enjoy both of those subjects. In particular, applicants should already have a background in those subjects.

   This project can accept up to two students.

2. *Mathematical Analysis of Problems in the Natural Sciences* by Vladimir Zorich

   The book covers three topics, "Analysis of Dimensions of Physical Quantities," "Multidimensional Geometry and Functions of a Very Large Number of Variables," "Classical Thermodynamics and Contact Geometry." Our goal would be to read and understand at least one of these (of the student's choice).

   This project only requires knowledge of calculus. Linear algebra would be even better, but is not required.

   This project will accept one student.

3. *Visual Complex Analysis* by Tristan Needham

   The primary goal for the project is to use complex analysis to study spherical and hyperbolic geometry. An essential ingredient will be the geometry of Möbius transformations, which are symmetries that arise all over complex analysis, hyperbolic geometry, and physics.

   We'll make our way through the text following the "Non-Euclidean Course" syllabus laid out in the author's preface, which contains sections of chapters 1,2, 3, 4, and 6. Ideally, we'd read a section or two each week, and we can skip around based on our interests.

   Suitable applicants will have had previous exposure to some proof-based real analysis (e.g. uniform convergence, series/sequences of numbers and functions) and multivariable calculus (e.g. line integrals). The most important prerequisite is that you must enjoy thinking geometrically.

   This project will accept one student.

4. *Representation Theory: A First Course* by W. Fulton & J. Harris

The project has two main goals. The first is to learn/review the basic tools of representation theory, which classically studies groups via their linear actions on vector spaces. The second is to study the interplay between representation theory and combinatorics, beginning with representations of the symmetric group and Young tableaux, and then continuing with more examples as time allows.

We will aim to cover Chapters 1-4 of Fulton-Harris, "Representation theory," available to Brown students via SpringerLink. We will supplement Chapter 4 with Fulton's book "Young tableaux," also available on SpringerLink. My preliminary estimate is that we should expect to spend 4-5 hours a week on this project; 3-4 hours reading and working on exercises, and another hour meeting as a group. By the end of the course, students will be prepared to read contemporary papers on representation theory.

Applicants should have some background in abstract algebra, e.g. Brown's course Math 1530.

This project will accept up to two applicants.

5. *Thirty-Three Miniatures* by Jiri Matousek

The goal of this project is to understand some linear-algebraic methods through interesting applications in combinatorics, geometry, and computer science.

The plan is to get through any 13 chapters of the textbook (in any order). By default, we could plan on sticking to the first 13, but the students can skip around (and/or do more than 13) if they find other applications more interesting.

This DRP would be best suited for students who are still trying to figure out if they'd like to pursue math. The project won't be super intense; the point is just to expose students to cool applications each week.

This project will accept up to four students.

6. *Set Theory* by William Weiss

We'll learn axiomatic set theory (ZFC), beginning with predicate logic and the language of set theory. Weiss' preprint will be supplemented by material from Devlin's *The Joy of Sets*. We will carefully prove such elementary theorems as "the successor to every natural number is a natural number."

Because set theory is foundational, no specific prerequisites are necessary. But this will be a very abstract and perhaps at times challenging project, so a familiarity with proofs and a willingness to grapple with abstraction will be helpful.

This project will accept one student.