Brown University
Mathematics Department
Course Syllabi

Version 0.95 — July 2005

Contents

Introduction

This booklet includes sample syllabi for all of the upper level (non-calculus) courses taught by the Mathematics Department at Brown University. It was created as a collaborative effort involving the entire department. It should be noted that the syllabi and lists of topics are merely guidelines, they are not meant to be prescriptive. Faculty at Brown are encouraged to teach subjects from their own perspective. This may involve inclusion of additional topics, substitution of new topics in place of those listed here, or exclusion of some topics in order to permit other areas to be studied at a greater depth. However, it is hoped that the publication of this set of guidelines will encourage some curricular standardization, without stifling creativity, and especially that it will be a useful resource for anyone teaching a particular course at Brown for the first time.

Joseph H. Silverman, Chair
November 2004

Math 42
Introduction to Number Theory

Course Catalog Description

An overview of one of the most beautiful areas of mathematics. Ideal for any student who wants a taste of mathematics outside of, or in addition to, the calculus sequence. Topics include: prime numbers, congruences, quadratic reciprocity, sums of squares, Diophantine equations, and, as time permits, such topics as cryptography and continued fractions.

Prerequisites

There are no prerequisites for Math 42, other than high school algebra.

Syllabus

Standard Topics.

- Pythagorean triples
- Divisibility, greatest common divisor, Euclidean algorithm, Fundamental Theorem of Arithmetic
- Congruences, solutions to linear congruences, Fermat's little theorem ($a^{p-1} \equiv 1 \bmod p$), Euler's formula ($a^{\phi(m)} \equiv 1 \bmod m$)
- Primes, infinitude of primes, Mersenne primes, perfect numbers
- Powering algorithm modulo m, computing roots modulo m, RSA public key cryptosystem
- Primitive roots modulo p, application to public key cryptography (Diffie-Hellman key exchange)
- Squares modulo p, quadratic character of -1 and of 2 (with proof), quadratic reciprocity (without proof, with applications)
- Sums of two squares

Additional Topics Selected From:

- Proof of quadratic reciprocity
- Square-triangular numbers (Pell's equation $x^2 - 2y^2 = 1$)
- Pell's equation $x^2 - Dy^2 = 1$
- Fermat equation $x^4 + y^4 = z^4$
- Carmichael numbers and primality testing
- Gaussian integers
- Irrational numbers and transcendental numbers

- Diophantine approximation
- Fibonacci numbers and linear recurrences
- Sums of powers and generating functions
- Elliptic curves
- Riemann zeta function and other analytic functions
- Solutions to Diophantine equations modulo p

Sample Textbooks

- *A Friendly Introduction to Number Theory*, J.H. Silverman, Prentice Hall (February 15, 2001), 383 pages.

Last Updated May 2003

Math 52
Linear Algebra

Course Catalog Description

Vector spaces, linear transformations, matrices, systems of linear equations,bases, projections, rotations, determinants, and inner products. Applications may include differential equations, difference equations, least squares approximations, and models in economics and in biological and physical sciences. MA 52 or MA 54 is a prerequisite for all 100-level courses in Mathematics except MA 126.

Prerequisites

MA 10, 17, or 19. May not be taken in addition to MA 54.

Syllabus

Remark. Math 52 should emphasize concrete topics and applications suitable for students interested in areas such as biology, computer science, and economics. We recommend that potential mathematics concentrators take Math 54, which gives includes an more thorough introduction to abstract vector spaces, linear transformations, and proofs.

Standard Topics.
Linear Equations. (*about 1.5 weeks*).

- Solution of n equations in n unknowns, matrices, matrix multiplication.
- Gaussian elimination, row operations, elementary matrices.
- LDU factorization.
- Matrix inversion by Gauss-Jordan algorithm.
- Computer science issues: algorithms, running time estimates.

Vector Spaces. (*4 weeks*).

Note: The more abstract concepts in this unit are difficult for many students. It is inadvisable to start the course with them.

- Solving $Ax = b$ for an $n \times n$ matrix A of rank r.
- General definitions: vector space, subspace, linear independence, basis, dimension.
- Examples of vector spaces: column vectors, matrices, polynomials, functions.
- Linear independence and dimension via Gaussian elimination, pivots, echelon form.

Geometry of Vector Spaces. (about 1.5 weeks).

- Lengths (norms), dot products, angle between vectors.
- Orthogonality, orthogonal projections, application to least squars.
- Gram-Schmidt orthogonalization (yielding another matrix factorization).

Determinants. (1.5 weeks).

- Determinant as multilinear alternating normalized function of n vectors in $\mathbb{R}^n$.
- Formulas for determinants.
- Cofactor matrix.

Eigenvalues and Eigenvectors. (3.5 weeks).

- Eigenvalues and eigenvectors.
- Jordan normal form.
- Applications chosen from:
  - Difference equations and linear recurrences.
  - Markov matrices, examples from economics or biology.
  - Differential equations $dx/dt = Ax$ with $A \in M_n(\mathbb{R})$.
  - Differential equations $dx/dt = A(t)x$.
  - Game theory.
  - Networks.
  - Lattices and lattice problems (SVP, CVP).

Additional Topics (as time permits).

- Inner products and bilinear forms.
- Symmetric, Hermitian, and unitary matrices.
- Statistics: Mean, Variance, Covariance, Regression

Sample Textbooks

- *Introduction to Linear Algebra*, Gilbert Strang, Wellesley Cambridge Press; (March 1, 2003).
- *Linear Algebra and Its Applications*, David C. Lay, Addison Wesley; (July 18, 2002), 576 pages

Last Updated April 2003

Math 54
Honors Linear Algebra

Course Catalog Description

Linear algebra for students of greater aptitude and motivation, especially mathematics and science concentrators with a good mathematical preparation. Matrices, linear equations, determinants, and eigenvalues; vector spaces and linear transformations; inner products; Hermitian, orthogonal, and unitary matrices; and Jordan normal forms. Provides a more extensive treatment of the topics in MA 52.

Prerequisites

MA 18, 20, or 35.

Syllabus

The syllabus for Math 54 is quite similar to Math 52, but with much more emphasis on general vector space concepts and on proofs. By the end of Math 54, students are expected to be able to devise and clearly explain simple proofs on their own. Math 54 makes time for the additional material and the added emphasis on proofs by moving somewhat faster and doing fewer examples in class than in Math 52.

Last Updated April 2003

Math 101
Introduction to Analysis

Course Catalog Description

Completeness properties of the real number system, topology of the real line. Proof of basic theorems in calculus, infinite series. Topics selected from ordinary differential equations. Fourier series, Gamma functions, and the topology of Euclidean plane and 3-space.

Prerequisites

A solid understanding of calculus and a liking for abstract mathematics. MA 18, 20, or 35. MA 53 or 54 may be taken concurrently. Most students ar advised to take MA 101 before MA 113.

Syllabus

Goals.

(I) Understanding logical constructions
(II) Practice in constructing mathematical proofs.
(III) Understanding calculus from a mathematical and logical point of view.
(IV) Leaning basic analysis.

Syllabus.

(1) Mathematical reasoning
(2) The real number system $\mathbb{R}$
(3) Topology of $\mathbb{R}$
(4) Continuous functions of one variable
(5) Differentiability of functions
(6) Integrability of functions (the Riemann integral)
(7) Sequences of functions and their limits

Detailed List of Standard Topics.

- Methods of Proof: Building proofs statements, connectives, proof by contradiction, proof by cases, open statements and quantifiers, proving universal statements, proving existential statemetns, negating a quantified statements.
- Set theory, cardinality, countable and uncountable sets.
- The algebra of real numbers, the rules of arithmetic, fields.
- Ordering, well-ordering, the natural numbers, and induction.

- Ordered fields, absolute value, distance, intervals, neighborhoods.
- Upper and lower bounds, suprema, the least upper bound axiom.
- Nested intervals, the archimedean property, decimal expansions.
- Cluster points, derived sets, the Bolzano-Weierstrass theorem.
- Topology of the real numbers, open sets, closed sets, direct an inverse images, continuous functions.
- Sequences: convergence, subsequences, monotone sequences, recursively defined sequences, the Bolzano-Weierstrass theorem, Cauchy sequences.
- Compace sets, the extreme value theorem, the covering property, the Heine-Borel theorem.
- Connected sets, the intermediate value theorem, disconnections, continuous functions and connected sets.
- Uniform continuity.
- Sequences and series of functions, pointwise convergence, uniform convergence, the Arzela-Ascoli theorem, the Weierstrass M-test, power series.
- Integration, upper and lower Riemann integrals, oscillations, integrability of continuous functions, the fundamental theorem of calculus.

Additional Topics (as time permits).

- Fourier series
- Gamma and other special functions
- Ordinary differential equations
- Topology of $\mathbb{R}^2$ and $\mathbb{R}^3$

Sample Textbooks

- *The Way of Analysis*, Robert S. Strichartz, Jones & Bartlett Publishers; Revised edition (June 1, 2000), 739 pages.
- *Introduction to Real Analysis*, Michael J. Schramm, Prentice Hall (August 8, 1995), 368 pages.
- *Introduction to Analysis*, Maxwell Rosenlicht, Dover Publications; (February 1, 1985), 254 pages.

Last Updated June 2003

Math 104
Introduction to Geometry

Course Catalog Description

Topics are chosen from Euclidean, non-Euclidean, projective, and affine geometry. Highly recommended for students who are concidering teaching high school mathematics.

Prerequisites

MA 52, 54, or permission of the instructor. A solid understanding of linear algebra and a liking for abstract mathematics.

Syllabus

Traditionally, all mathematics has been divided into three parts: algebra, analysis, and geometry. All three appear in the classic work of Euclid, although the algebra and the analysis are always expressed in geometric language.

For two millenia Euclid reigned supreme as the first (and only) axiomatic/deductive mathematical system, its only "flaw" being the verbose, and apparently unnecessary, "fifth postulate" on parallels. Ironically, Euclid was vindicated at the very moment he was dethroned: His fifth postulate is indeed necessary, yet there are other geometric systems that are just as logically consistant as Euclid's.

The first such system was the hyberolic geometry of Bolyai and Lobachevshy, followed by the elliptic geometry of Riemann. These were unified under the banner of projective geometry by Cayley and culminated in Klein's Erlanger Programm (geometry equals the invariants of a group of transformations) in which Euclid is reversed: geometry becomes subservient to algebra. Math 104 is an introduction to some of these ideas.

Standard Topics.

- The affine and projective planes
- The geometry of curves in the projective plane
- The geometry of surfaces in projective space
- An introduction to some abstract concepts (equivalence relations, groups, division rings)

Detailed list of topics.

- The affine plane
- Points at infinity and the projective plane
- Duality
- The cross-ratio
- Inversion
- Singularities
- Bezout's theorem on intersecting curves
- Desargues' theorem on triangles
- Pascal's theorem on hexagons
- Poncelet's closure theorm
- The group law on a cubic curve
- The 27 lines on a cubic surface

Sample Textbooks

- *Euclidean & Non-Euclidean Geometry : Development and History*, Marvin Jay Greenberg, W. H. Freeman; 3rd edition (July 15, 1993), 512 pages.
- *Projective Geometry and Modern Algebra*, Lars Kadison, Birkhäuser (1996), 208 pages.
- *Geometry Revisited*, H. S. M. Coxeter, Samuel L. Greitzer, The Mathematical Association of America (September 5, 1996), 207 pages.

Last Updated June 2003

Math 106
Differential Geometry

Course Catalog Description

The study of curves and surfaces in 2- and 3-dimensional Euclidean space using the techniques of differential and integral calculus and linear algebra. Topics include curvature and torsion of curves, Frenet-Serret frames, global properties of closed curves, intrinsic and extrinsic properties of surfaces, Gaussian curvature and mean curvature, geodesics, minimal surfaces, and the Gauss-Bonnet theorem.

Prerequisites

Standard prerequisites for 100 level courses, that is, calculus up through multivariable calculus (one of MA 18, 20, or 35) and one semester of linear algebra (MA 52 or 54).

Syllabus

Mathematics 106 treats differential geometry of curves in the plane of curves and surfaces in three-space. The course falls into three parts, of roughly equal size.

1. Geometry of Curves in the Plane.

- Arclength, tangential and normal vectors, (signed) curvature.
- Reconstruction of a curve with given curvature and arclength.
- Evolutes and involutes.
- Global results such as the four-vertex theorem, the isoperimetric inequality, and Hopf's theorem on the tangential degree of an embedded closed curve.

1. Geometry of Curves in the Space.

- Arclength, curvature, torsion, and the Frenet-Serret equations.
- Reconstruction of a curve with given curvature and torsion.
- Generalized helices.
- Evolutes and involutes.
- Global results such as Fenchel's theorem and the Fary-Milnor theorem on the total curvature of knotted curves.

3. Surfaces in Space.
- The first and second fundamental forms.
- Area and the Gauss and Codazzi equations.
- Gaussian curvature, developable surfaces, principal curvature, Meunier's Theorem, surfaces of constant Gaussian curvature, mean curvature, minimal surfaces.

4. Intrinsic Geometry of Surfaces.
- The Theorema Egregium.
- Geodesic curvature of curves on surfaces.
- First variation of arclength.
- The Gauss-Bonnet Theorm and applications (both intrinsic and extrinsic), such as hyperbolic geometry in the upper half plane and the degree of the spherical image mapping.

Sample Textbooks

- *Lectures on classical differential geometry*, Dirk Jan Struik, Dover Publications; 2nd ed. edition (May 1, 1988), 240 pages.

Last Updated June 2003

Math 111
Ordinary Differential Equations

Course Catalog Description

Ordinary differential equations, including existence and uniqueness theorems and the theory of linear systems. Topics may also include stability theory, the study of singularities, and boundary value problems.

Prerequisites

Standard prerequisites for 100 level courses, that is, calculus up through multivariable calculus (one of MA 18, 20, or 35) and one semester of linear algebra (MA 52 or 54).

Syllabus

Standard Topics.

- Intuition on ODE's, first-order equations, scientific applications (1.5 weeks).
- Linear constant-coefficient $n \times n$ systems in general, exponential of a matrix, reduction of high-order systems to first-order. (2 weeks).
- Phase plane portraits of 2×2 systems. (1 week)
- Variable-coefficients, Wronskian, power series solutions, Bessel and other special equations (1 week).
- Nonlinear systems, general construction of solutions (Born approximation), existence and uniqueness (1 week).

Additional Topics (two or more, as time permits).

- Nonlinear systems, linearization, nodes, saddles, etc., phase portraits (1.5 weeks).
- Applications to ecology, mechanics, physiology, etc. (1 week).
- Numerical computation, Euler, Runga-Kutta (1 week).

Sample Textbooks

- *Differential Equations: A Modern Approach*, Harry Hochstadt, Dover Publications; (August 1, 1975), 294 pages.
- *Ordinary Differential Equations* (Chapman Hall/CRC Mathematics), D. K. Arrowsmith, C. M. Place, Chapman & Hall; (March 1, 1982), 252 pages

- *Nonlinear Ordinary Differential Equations: An Introduction to Dynamical Systems* (Oxford Applied and Engineering Mathematics), D. W. Jordan, Peter Smith, Oxford University Press; 3rd edition (August 1, 1999), 550 pages.

Last Updated November 2004

Math 112
Partial Differential Equations

Course Catalog Description

The wave equations, the heat equation, Laplace's equation, and other classical equations of mathematical physics and their generalizations. Solutions in series of eigenfunctions, maximum principles, the method of characteristics, Green's functions, and discussion of well-posedness.

Prerequisites

Standard prerequisites for 100 level courses, that is, calculus up through multivariable calculus (one of MA 18, 20, or 35) and one semester of linear algebra (MA 52 or 54).

Syllabus

Standard Topics.

- Types of PDE's, solving simple equations by characteristics (1 week).
- Applications to vibrating string, heat, fluids, diffusion, quantum mechanics; boundary conditions (1 week).
- Solution of one-dimensional wave equations and heat equations (1.5 weeks).
- Solution of boundary problems (Dirichlet, Newmann, Robin) by Fourier series (2 weeks).
- Completeness and convergence of Fourier series, Gibbs' phenomenon (1.5 weeks).
- Laplace's equation in rectangles, circles, wedges, etc., maximum principle, polar coordinates, introduction to Green's functions (2 weeks).
- Numerical computation of solutions by finite differences and finite elements (2 weeks)
- Solution of the three-dimensional wave, heat, and Maxwell equations (1.5 weeks).

Additional Topics (as time permits).

- Distributions and Green's functions.
- Fourier and Laplace transforms.
- Eugenvalue problems in general domains.
- Further applications to physics.

- Introduction to nonlinear PDE's.

Sample Textbooks

- *Partial Differential Equations*: *An Introduction*, Walter Strauss, Wiley; (March 3, 1992), 440 pages.

Last Updated November 2004

Math 113–114
Functions of Several Variables

Course Catalog Description

A course on calculus on manifokds. Included are differential forms, integration, and Stokes' formula on manifolds, with applications to geometrical and physical problems, the topology of Euclidean spaces, compactness, connectivity, convexity, diffferentiability, and Lebesgue integration.

Prerequisites

Standard prerequisites for 100 level courses, that is, calculus up through multivariable calculus (one of MA 18, 20, or 35) and one semester of linear algebra (MA 52 or 54). It is recommended that a student take a 100-level course in analysis before attempting MA 113.

Syllabus

Math 113.

- Countable and uncountable sets.
- Axioms for the real line and basic properties (briefly).
- Euclidean space, metric spaces and completeness.
- Topological spaces, continuity, compactness, connectedness and the Cantor set.
- Examples of function spaces, Picard and Peano Theorems.
- Measures, outer measures and the construction of Lebesgue measure.
- Lebesgue integral and convergence theorems, L^p spaces, Fubini's theorem.

Math 114.

- Differentiability in Euclidean space, some multilinear algebra.
- Inverse and implicit function theorems.
- Jacobians.
- Manifolds as subsets of $\mathbb{R}^n$, tangent spaces and orientability.
- Embeddings, immersions and regular values.
- Exterior product, exterior derivative and pullback.
- Differential forms, tensor fields.
- Integration of forms and Stokes' Theorem.

Sample Textbooks

- *Mathematical Analysis: An Introduction* (Undergraduate Texts in Mathematics), Andrew Browder, Springer-Verlag; (January 1, 1996), 333 pages.
- *Functions of Several Variables* (Undergraduate Texts in Mathematics), Wendell Fleming, Springer-Verlag; 2nd edition (February 1, 1997), 411 pages.
- *Principles of Mathematical Analysis*, Walter Rudin, McGraw-Hill Science/Engineering/Math; 3rd edition (January 1, 1976), 325 pages, ISBN: 0070856133, 007054235X.

Last Updated June 2003

Math 126
Complex Analysis

Course Catalog Description

Examines one of the cornerstones of mathematics. Complex differentiability, Cauchy-Riemann differential equations, contour integration, residue calculus, harmonic functions, geometric properties of complex mappings.

Prerequisites

Calculus up through multivariable calculus (one of MA 18, 20, or 35). This course does *not* require linear algebra (MA 52 or 54).

Syllabus

Standard Topics.

- Complex numbers, the complex expoential, powers and roots.
- Analytic functions, the Cauchy-Riemann equations, harmonic functions.
- Elementary functions, exponential, trigonometric, and hyperbolic functions, the logarithm function, complex powers, inverse trigonometric functions.
- Complex integration, contour integals, independence of path, Cauchy's integral theorem and its consequences, bounds for analytic functions, applications to harmonic functions.
- Series representations for analytic functions, sequences and series, Taylor series, power series, Larent series, zeros and singularities, the point at infinity, analytic continuation.
- Residue theory, trigonometric integrals over $[0, 2\pi]$, improper integrals of certain functions over $(-\infty, \infty)$, improper integrals involving trigonometric functions, indented contours, integrals involving multiple-valued functions, the argument principle and Rouche's theorem.
- Conformal mappings, invarience of Laplace's equation, geometric considerations, Mobius transformations, the Schwarz-Christoffel transformation.

Additional Topics (as time permits).

- Entire and meromorphic functions on the plane.
- Partial fraction decompositions
- Infinite products

- The Gamma function.
- Analytic number theory (ζ-function, L-series)

Sample Textbooks

- *Fundamentals of Complex Analysis with Applications to Engineering, Science, and Mathematics*, Edward B. Saff, Arthur D. Snider, Prentice Hall; 3rd edition (December 31, 2002), 563 pages.
- *Complex Analysis*, T.W. Gamelin, Springer-Verlag; 1st edition (May 18, 2001), 464 pages.
- *Complex Analysis* (Princeton Lectures in Analysis), Elias M. Stein, Rami Shakarchi Princeton University Press (April 7, 2003), 392 pages, ISBN: 0691113858.

Last Updated June 2003

Math 127
Topics in Functional Analysis

Course Catalog Description

Infinite-dimensional vector spaces with applications to some or all of the following topics: Fourier series and integrals, distributions, differential equations, integral equations, calculus of variations.

Prerequisites

At least one 100-level course in Mathematics or Applied Mathematics, or permission of the instructor, with MA 101 being recommended. This course assumes no measure theory.

Syllabus

The following syllabus is a version of Math 127 that that emphasizes Fourier analysis and its applications.

Standard Topics.

- Basic Properties of Fourier Series: pointwise convergence, summability methods, Poisson kernel, approximation of continuous functions by trigonometric polynomials.
- Mean square convergence, vector spaces of functions, inner products.
- Applications of Fourier series (see chapter 4 of Stein-Shakarchi and Dym-McKean for applications in geometry, partial differential equations, probability, and number theory).
- The Fourier transform on the real line: inversion, Plancherel, applications to PDE (heat equation), Poisson summation.

Additional Topics (as time permits).

- Discrete Fourier Analysis, analysis on groups
- Wavelets and other bases of L^2.
- Shannon sampling, multiresolution analysis, and applications to signal processing and compression.

Sample Textbooks

- *Fourier Analysis: An Introduction* (Princeton Lectures in Analysis, Volume 1), Elias M. Stein, Rami Shakarchi, Princeton University Press (March 17, 2003), 320 pages.

- *Fourier Series and Integrals* H. Dym & H.P. McKean, (Probability and Mathematical Statistics, Vol 14), Academic Press; (October 28, 1985), 295 pages.

Last Updated June 2003

Math 141
Combinatorial Topology

Course Catalog Description

Topology of Euclidean spaces, winding number and applications, knot theory,fundamental group and covering spaces. Euler characteristics, simplicial complexes, classification of two-dimensional manifolds, vector fields, the Poincare-Hopf theorem, and introduction to three-dimentional topology.

Prerequisites

Standard prerequisites for 100 level courses, that is, calculus up through multivariable calculus (one of MA 18, 20, or 35) and one semester of linear algebra (MA 52 or 54).

Syllabus

Math 141 is an introduction to some of the main concepts, problems and results in topology (also called "analysis situs" or "rubber-sheet geometry"). Traditional Euclidean geometry considers two figures to be the same when they are congruent, i.e., if they can be mapped to one another by a rotation, reflection or translation (or combination thereof). In topology, however, figures are topologically eqivalent (or "homeomorphic") if they can be mapped to one another by any continuous function. So a topologist cannot tell the difference between a coffee cup and a doughnut, but can distinguish both of them from a pretzel. This is because the number of "holes" in an object is a "topological invariant," i.e., it doesn't change under a homeomorphism. The two main (and closely related) problems of topology are:

– Classify all topological spaces up to homeomorphism.
– Find a complete set of topological invariants.

Math 141 considers some aspects of these (still unsolved) problems.

Standard Topics.

- One dimensional topology: graphs and knots
- Planarity, colorability, winding number
- Topology of surfaces
- Euler characteristic, orientability, classification of surfaces, homology groups.

- Jordan curve theorem, Kuratowski's embedding theorem, the Borsuk-Ulam theorem, Brouwer's fixed point theorem, and the Poincare index theorem.
- Continuity, connectedness, compactness and completeness.
- Klein bottles, horned spheres monkey saddles.

Sample Textbooks

- *Combinatorial Introduction to Topology*, Michael Henle, W H Freeman & Co (December 1, 1978), reprinted by Dover Publications (March 14, 1994), 310 pages.
- *Topology* (2nd Edition), James Munkres Prentice Hall; 2nd edition (December 28, 1999), 537 pages, ISBN: 0131816292.

Last Updated June 2003

Math 153
Abstract Algebra

Course Catalog Description

An introduction to the principles and concepts of modern abstract algebra. Topics include groups, rings, and fields; applications to number theory, the theory of equations, and geometry. MA 153 is required of all students concentrating in mathematics.

Prerequisites

Standard prerequisites for 100 level courses, that is, calculus up through multivariable calculus (one of MA 18, 20, or 35) and one semester of linear algebra (MA 52 or 54).

Syllabus

Standard Topics.
Groups.

- Subgroups
- Normal subgroups
- Homomorphisms
- Cyclic groups
- Abelian groups
- Structure theorem for finite abelian groups (statement)
- Direct products
- Semi-direct products
- Isomorphism theorems
- Sylow theorems (statements and applications)
- Jordan-Hölder theorem
- Symmetric and alternating groups
- Classification of groups of small order

Rings.

- Ideals
- Homomorphisms
- Commutative rings
- Integral domains
- PID's, UFD's, Euclidean rings, R is UFD implies $R[X]$ is UFD
- Primes ideals and maximal ideals

Fields.

- Finite and algebraic extensions
- Minimal polynomials
- Finite fields
- Impossibility of trisecting angles

Additional Topics (as time permits).

- Localization of integral domains (e.g. fields of fractions)
- Proofs of Sylow theorems
- Categories and functors
- Norms and traces

Sample Textbooks

- *Abstract Algebra*, David S. Dummit & Richard M. Foote, Wiley; (July 11, 2003), 944 pages.
- *Topics in Algebra*, I. Herstein, Wiley; (June 6, 1975), 400 pages.

Last Updated June 2003

Math 154
Topics in Abstract Algebra

Course Catalog Description

Galois theory together with topics in algebra. Examples of subjects which have been presented in the past include algebraic curves, group representations, and the advanced theory of equations.

Prerequisites

MA 153.

Syllabus

Math 154 generally covers Galois theory together with one or more special topics. Math 154 may be repeated for credit.

Standard Topics.

- Review field theory
- Splitting fields and normal extensions
- Cyclotomic fields
- Theorem of the primitive element
- Fundamental theorem of Galois theory
- Applications of Galois theory (one or more)
  - Fundamental theorem of algebra
  - Solvable groups and solvable extensions
  - Insolubility of the quintic (and higher degree polynomials)
  - Explicit solution of the cubic
- Norm and trace

Additional Topics (as time permits). There are many possible special topics that are appropriate for Math 154. Some topics that have been covered in recent years include the following:

- Algebraic geometry
- Representations of finite groups
- Categories and functors
- Elliptic curves
- Algebraic number theory

Sample Textbooks

Same as for Math 153.

Last Updated June 2003

Math 156
Number Theory

Course Catalog Description

A basic introduction to the theory of numbers. Unique factorization, prime numbers, modular arithmetic, quadratic reciprocity, quadratic number fields, finite fields, diophantine equations, and related topics.

Prerequisites

The principal prerequisites for Math 156 are Math 52 or 54 (linear algebra) and Math 153 (groups/rings). Students should know the definition of a field, but they are not expected to know about extension fields or Galois theory. Most will not have taken complex analysis.

Syllabus

Standard Topics.

(1) Divisibility
  - Greatest common divisor and the Euclidean algorithm
  - Fundamental Theorem of Arithmetic ($\mathbb{Z}$ is a UFD)

(2) Congruences
  - Solution of linear congruences
  - Chinese remainder theorem
  - Fermat's little theorem ($a^{p-1} \equiv 1 \bmod p$)

(3) Arithmetic functions
  - Euler's ϕ, number of divisors, sum of divisors
  - Euler's formula ($a^{\phi(m)} \equiv 1 \bmod m$)
  - Multiplicative functions, Dirichlet product, Möbius inversion formula

(4) Prime numbers
  - Infinitude of primes (with congruence conditions)
  - Estimates for $\pi(x)$
  - Mersenne primes and application to perfect numbers

(5) Primitive element theorem for $\mathbb{F}_p$

(6) Quadratic reciprocity
  - Legendre and Jacobi symbols
  - Euler's criterion ($a^{(p-1)/2} \equiv \left(\frac{a}{p}\right) \bmod p$) and Gauss' criterion ($(-1)^{\mu} = \left(\frac{a}{p}\right)$)
  - Proof of quadratic reciprocity

(7) Diophantine equations, one of more of the following:
- Linear equations $ax + by = c$
- Pell's equation $x^2 - Dy^2 = 1$
- Fermat's equation $x^4 + y^4 = z^4$
- Homogeneous equations $f(x, y) = a$ and Diophantine approximation

(8) Analytic number theory, one or more of the following:
- Riemann ζ function, Euler product, analytic continuation (at least to $s > 0$)
- Special values $\zeta(2k)$ and Bernoulli numbers
- Average values of arithmetic functions
- Characters and L-series

Additional Topics (as time permits).

(1) Finite fields $\mathbb{F}_q$ (existence, cyclicity of $\mathbb{F}_q^*$)

(2) Gaussian integers
- Unique factorization
- Description of primes
- Application to sums of two squares

(3) Quadratic number fields
- Units (via Pell's equation)
- Examples of non-unique factorization
- Unique factorization of ideals

(4) Cryptography (RSA and/or discrete logarithm based systems)

(5) Higher order reciprocity laws
- Gauss sums and Jacobi sums
- Cubic reciprocity and $\mathbb{Z}[\rho]$
- Quartic reciprocity

(6) Elliptic curves (over $\mathbb{Q}$ and/or over $\mathbb{F}_p$)

(7) Solutions to Diophantine equations over $\mathbb{F}_p$ (or $\mathbb{F}_q$)
- Upper bounds for $\#X(\mathbb{F}_p)$
- Zeta functions and Weil conjectures

(8) Solving congruences modulo p^n and Hensel's lemma

(9) Additive number theory
- Partitions
- Sums of (almost) primes and Goldbach's conjecture

(10) Computational number theory
- Primality testing
- Factorization algorithms (quadratic sieve)
- Discrete logarithm algorithms (baby step-giant step, index calculus)
- Resultants and discriminants

Sample Textbooks

- *A Classical Introduction to Modern Number Theory*, K. Ireland and M. Rosen, Graduate Texts in Mathematics 84, Springer-Verlag; 2nd edition (January 1, 1991), 408 pages.
- *Introduction to Analytic Number Theory*, T. Apostol, Undergraduate Texts in Mathematics, Springer-Verlag; Corr. 5th print edition (June 1, 1976), 352 pages.

Last Updated May 2003

Math 158
Cryptography

Course Catalog Description

Topics include symmetric ciphers, public key ciphers, complexity, digital signatures, applications and protocols.

Prerequisites

Linear algebra (MA 52 or MA 54). Abstract algebra (MA 153) is *not* required for the course, the necessary material will be covered in class.

Syllabus

Standard Topics.

Introduction to Cryptology: Substitution ciphers, mathematical preliminaries (divisibility, modular arithmetic, finite fields, counting arguments), symmetric and asymmetric ciphers

Discrete Logarithms and Diffie-Hellman: The discrete logarithm problem, Diffie-Hellman key exchange, ElGamal public key cryptosystem, Chinese Remainder Theorem, Pohlig-Hellman algorithm for DLP

Probability Theory and Information Theory: Probabilty theory, collision algorithms, information theory and entropy

Integer Factorization and RSA: More math (Euler's theorem, roots modulo pq), RSA public key cryptosystem, primality testing, the $p-1$ factorization method, smooth numbers and sieves, quadratic residues and quadratic reciprocity, probabilistic encryption

Elliptic Curves and Cryptology: geometry and algebra of elliptic curves, elliptic curves over finite fields, the elliptic curve discrete logarithm problem (ECDLP), elliptic curve cryptography, Pollard's ρ method

Lattices and Cryptology: Math review (vector spaces and linear algebra), integer lattices and the shortest vector problem, applications of lattice reduction to cryptanalysis, more math (polynomial rings, quotient rings, and convolutions), NTRU public key cryptosystem

Digital Signatures and Other Constructions: Digital signatures, hash functions, random numbers and pseudorandom number generators

Additional Topics (as time permits).

- Identification schemes
- Zero-knowledge proofs
- The elliptic curve factorization algorithm
- Finite fields with p^k elements
- The LLL lattice reduction algorithm
- Knapsack public key cryptosystems
- Discrete logarithm digital signatures and DSA
- NTRU digital signatures
- Building protocols from cryptographic primitives
- Quantum cryptography and quantum computing

Sample Textbooks

- *Making, Breaking Codes: Introduction to Cryptology*, Paul Garrett, Prentice Hall; 1st edition (August 9, 2000), 525 pages.
- *A Mathematical Introduction to Cryptography*, J. Hoffstein, J. Pipher, J.H. Silverman, preprint.
- *Cryptography: Theory and Practice*, Douglas Stinson, Chapman & Hall/CRC; 2 edition (February 27, 2002), 360 pages.

Last Updated November 2004

Math 161
Probability

Course Catalog Description

Basic probability theory. Sample spaces; random variables; normal, Poisson, and related distributions; expectation; correlation; and limit theorems. Applications in many fields (biology, physics, gambling, etc.).

Prerequisites

Math 18 and 52 or the equivalent. A firm grasp of algebra, analytic geometry and calculus and a willingness to think mathematically are also essential.

Syllabus

Standard Topics. This course is an introduction to some of the main ideas and techniques in the study of probability. Topics include:

- modeling uncertainty (sample/probability spaces)
- computing probabilities (set theory, combinations)
- random variables (discrete and continuous, density and distribution functions)
- expected value (mean/variance, moment generating functions, inequalities)
- joint distributions (independence, covariance, correlation)
- conditional probability/expectation (Bayes' rule, regression)
- asymptotic results (law of large numbers, central limit theorem)
- stochastic processes (Markov chains, Poisson processes)

Additional Topics (as time permits).

- inclusion-exclusion theorems
- Jensen's inequality
- score functions

Sample Textbooks

- *A First Course in Probability*, Sheldon Ross, Prentice Hall; 6 edition (July 31, 2001), 528 pages.

Last Updated June 2003

Math 162
Mathematical Statistics

Course Catalog Description

Central limit theorem, point estimation, interval estimation, multivariate normal distributions, test of hypotheses, and linear models.

Prerequisites

MA 161 or written permission. A solid foundation in probablility theory (equivalent to Math 161) is essential, as is skill in manipulating (and understanding) algebraic, calculus, and probabilistic expressions. Matrix algebra will occasionally be helpful.

Syllabus

The subject of Math 162 is statisical inference: the art (science?) of drawing conclusions from data and deciding whether they are "statistically significant" at an appropriate "confidence level."

- Reveiw of probability (conditional expectation, Jensen's inequality and the score functions).
- Four important distributions (normal, chi-square, t, and F).
- The vocabulary of statistics (population, random sample, estimator).
- Evaluating estimators (bias, efficiency, sufficiency, Cramer-Rao inequality, Rao-Blackwell theorem).
- Testing hypotheses (critical regions, erroe types, power, liklihood ratio, Neyman-Pearson lemma).
- Nonparametric statistics (sign test, rank test, Kolmogorov-Smirnov theorem).
- Linear models (regression, ANOVA, Gauss-Markov theorem).

Sample Textbooks

- *An Introduction to Mathematical Statistics and Its Applications*, Richard J. Larsen & Morris L. Marx, Prentice Hall; 3rd edition (January 15, 2000), 790 pages.
- *The Cartoon Guide to Statistics*, Larry Gonick, Woollcott Smith, HarperResource; 1st HarperPerennial edition (February 25, 1994), 230 pages. [Supplementary textbook]

Last Updated June 2003

Math 201
Diferential Geometry

Course Catalog Description

Introduction to differential geometry (differential manifolds, differential forms, tensor fields, homogeneous spaces, fiber bundles, connections, and Riemannian geometry), followed by selected topics in the field.

Prerequisites

MA 211

Syllabus

The purpose of Math 201 is to give an introduction to Riemann Geometry. The first part of the course covers the following topics. The second is based on a topic chosen by the instructor.

0. Overview of Curves and Surfaces
   - Frenet-Serret Equations
   - Euler's Theorem
   - Gauss Curvature and Gauss-Bonnet Theorem
1. Manifold Review
   - Basic definition, including bundles: vector, fiber, principle, examples.
   - Tangent and cotangent bundles
   - Form and tensor bundles and operations: wedge, d, pullback, contraction.
2. Metric basics
   - Metrics
   - Raising and lowering operations
   - Metric contraction
   - Volume form
3. Connections
   - Connections on a vector bundle.
   - Levi-Civitia connection, Christoffel symbols, indices, tensor derivations. Extending the connection to tensor bundles.
   - Covariant derivative, parallel transport.
   - Geodesics, Gauss lemma, normal neighborhoods, normal coords.
   - Completeness, Hopf-Rinow

4. Curvature
   - Basic definitions, properties, including the Bianchi identities.
   - Meaning of curvature: Vanishing implies locally isometric to $\mathbb{R}^n$; interpretation via limit of parallel transport around small curves.
   - Method of Moving Frames, structure equations.
   - Jacobi fields and geodesic variation; Conjugate points and Cartan-Hadamard. Fomula for the volume of geodesic balls involving the scalar curvature.
   - Space forms.
   - Ricci formula, Bochner identity, applications, including application to $b_1(M)$ with nonnegative Ricci, and Lichnerowicz lower bound of 1.
   - Variations of energy of a path and the Jacobi equation; Bonnet-Myers, Synge-Weinstein.
5. More Comparison Theorems
   - Index lemma and Rauch comparison.
   - Bishop-Gunther volume comparison.
   - Sphere theorem.
   - Hessian comparison.
6. Isometric Immersions
   - Second fund form, Gauss map; examples include surfaces in Euclidean spaces and discussion of Euler-Muesnier theorem, Gauss map and Gauss curvature.
   - The Normal connection; Gauss-Codazzi equations; constraint equations in Relativity.
   - Realizing sectional curvatures as the Gauss curvature of the toally geodesic surface gotten by exponentiating a two-plane.

Remark. The above material is mostly covered in DoCarmo, chapters 0–9, with supplements needed for connections on vector bundles, structure equations, and tensor computations with indices.

Additional Topics (as time permits).

- Characteristic classes and Chern-Weil, Chern-Bonnet theorem.
- Toponogov's theorem.
- Maximum Diameter Theorem.
- Cheeger-Gromoll Splitting theorem.
- Hodge Theorem.
- Gauge Theories.

- Kähler Geometry.
- Symmetric spaces.

Sample Textbooks

- *Riemannian Geometry*, Manfredo Do Carmo & Francis Flaherty, Birkhauser Boston (February 24, 1994), 320 pages.
- *Introduction to Smooth Manifolds*, John M. Lee, Springer (September 23, 2002), 648 pages, ISBN: 0387954481.

Last Updated June 2003

Math 205-206
Algebraic Geometry

Course Catalog Description

Complex manifolds and algebraic varieties, sheaves and cohomology, vector bundles, Hodge theory, Khler manifolds, vanishing theorems, the Kodaira embedding theorem, the Riemann-Roch theorem, and introduction to deformation theory.

Prerequisites

Algebra 251-252 is a must. Students should know manifolds, basics of general topology and complex analysis. π_1 and singular (co)homology are necessary for the part of the course treating cohomology of sheaves. It is strongly suggested that all graduate students specializing in Algebraic Geometry take both both topology cources (MA 241–242).

Syllabus

Standard Topics.

1. Basics.

- Affine, projective and quasiprojective varieties.
- Proper and finite maps.
- Dimension.
- Singular and nonsingular points.
- Normal varieties and normalization.
- Blow ups and resolution of singularities (example: surfaces).
- Divisors, Riemann-Roch for curves (no proof).
- Jacobians for complex curves, Abel's theorem.
- Intersection theory.

2. Sheaves.

- Functors between the categories of sheaves.
- Cohomology of sheaves, spectral sequences.
- Proof of De Rham theorem.
- Coherent sheaves.
- Proof of the Riemann-Roch theorem for curves.

3. Examples.

- Grassmannians, flag varieties, Schubert cells.
- Elliptic curves.
- Discussion of surfaces (if time permits)

4. Schemes.

- Proper and smooth morphisms of schemes.
- Chern classes, formulation of Grothendieck-Riemann-Roch theorem.

5. Complex algebraic geometry.

- Kahler varieties and their properties.
- Connections and characteristic classes (the Chern-Weil theory).
- Hodge theory — results.

Additional Topics (as time permits).

- Abelian varieties.
- Hodge theory.
- Étale cohomology.
- Derived categories and algebraic theory of D-modules.
- Diophantine geometry.

Sample Textbooks

- *Basic Algebraic Geometry I: Varieties in Projective Space*, I. R. Shafarevich & M. Reid, Springer-Verlag Berlin Heidelberg (June 1, 2002).
- *Schemes and complex manifolds*, I.R. Shafarevich, Second edition. Berlin, 1994.
- *Algebraic Geometry*, R. Hartshorne, Springer-Verlag (June 1, 1977), 512 pages .
- *The Red Book of Varieties and Schemes*: Includes the Michigan Lectures (1974) on *Curves and Their Jacobians* (Lecture Notes in Mathematics 1358), David Mumford, Springer-Verlag; 2nd Expand edition (December 15, 1999), 304 pages.
- *Principles of Algebraic Geometry*, P. Griffiths & J. Harris, Wiley-Interscience; (August 2, 1994), 832 pages.
- *Diophantine geometry: An introduction*, M. Hindry & J.H. Silverman, Graduate Texts in Mathematics 201, Springer-Verlag; 1st edition (January 15, 2000), 558 pages.

Last Updated June 2003

Math 211
Introduction to Manifolds

Course Catalog Description

Inverse function theorm, manifolds, bundles, Lie groups, flows and vector fields, tensors and differential forms, Sard's theorem and transversality, and further topics chosen by instructor.

Prerequisites

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Syllabus

This course is designed to be an elementary introduction to the theory of manifolds and related topics, such as Lie groups, vector bundles, elementary differential topology, etc.

Standard Topics.

- Inverse and implicit function theorems, the constant rank theorem.
- Manifolds with or without boundary.
- Submanifolds, general constructions of manifolds.
- Tangent vectors, multilinear algebra, vector bundles, tangent and cotangent bundles, tensor fields, differential forms, partitions of unity, Stokes' theorem.
- Flows on manifolds, the Lie derivative, the Frobenius theorem.

Additional Topics (as time permits).

- Elementary theory of Lie groups and examples, principle and associate bundles.
- Sard's theorem, embedding of compact manifolds in $\mathbb{R}^n$, elements of transversality theory, elements of intersection theory, Lefschetz fixed-point theory, the Poincaré-Hopf theorem.

Sample Textbooks

Different parts of the course follow different textbooks, such as the following:

- *Differentiable Manifolds*, Lawrence Conlon, Birkhauser Boston; 2nd edition (March 30, 2001), 432 pages.
- *Differential Topology*, Victor Guillemin & Alan Pollack, Prentice Hall; 1 edition (August 14, 1974), 222 pages.

- *Foundations of Differential Manifolds and Lie Groups*, F. Warner, Springer-Verlag (October 1, 1983), 292 pages.
- *Lectures on the Geometry of Manifolds*, Liviu I. Nicolaescu, World Scientific Publishing Company (December 1, 1996), 481 pages.

Math 221-222
Real Function Theory

Course Catalog Description

Point set topology, Lebesgue measure and integration, L^p spaces, Hilbert space, Banach spaces, differentiability, and applications.

Prerequisites

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Syllabus

Standard Topics — Math 221.

(1) Measure Theory
- σ algebras
- outer measures
- Borel measures
- Lebesgue measure
- Cantor sets

(2) Integration
- measurable functions
- modes of convergence
- product measures and n-dimensional integrals
  Note: It is important to cover general measure spaces, not just Euclidean spaces.

(3) Signed measures and Differentiation
- Radon-Nikodym theorem
- Lebesgue Differention theorem
- functions of bounded variation

(4) L^p spaces
- Chebychev's inequality
- Minkowski's inequality
- Holder's inequality (generalized Young's inequality)
- duality
- interpolation (if time permits)
- convolution of functions

Standard Topics — Math 222.

(1) Basics of Functional Analysis
- Banach spaces
- linear functionals

- Baire Category
- open mapping theorem
- closed graph theorem
- uniform boundedness principle
- the weak topology
- Alaoglu's weak compactness theorem

(2) Hilbert spaces
(3) Radon measures
(4) Riesz representation theorem
- the dual of the space of continuous functions

Additional Topics for MA 222 (as time permits).

- Fourier transform, Fourier inversion, Hausdorff-Young theorem, Sobolev spaces.
- Fourier series and summability methods, pointwise convergence of Fourier series, distribution, applications of FT to PDE.
- Distribution theory, Integral operators, Fredholm alternative.
- Differentiation theorems, area and co-area formulas, Hausdorff measure.
- Operator theory (self adjoint, compact operators), spectral theory.

Sample Textbooks

- For MA 221: *Measure and Integral*, Richard Wheeden, Antoni Zygmund, Marcel Dekker; (June 1, 1977), 288 pages.
- For MA 222: *Measure Theory and Fine Properties of Functions*, Lawrence C. Evans, Ronald F. Gariepy, CRC Press; (December 18, 1991), 288 pages.
- For MA 222: *Analysis*, Elliott H. Lieb, Michael Loss, American Mathematical Society; 2nd edition (April 1, 2001), 346 pages.
- *Real and Complex Analysis*, Walter Rudin, McGraw-Hill Science/Engineering/Math; 3 edition (May 1, 1986), 483 pages, ISBN: 0070542341.

Last Updated June 2003

Math 225-226
Complex Function Theory

Course Catalog Description

Introduction to the theory of analytic functions of one complex variable. Content varies somewhat from year to year, but always includes the study of power series, complex line integrals, analytic continuation, conformal mapping, and an introduction to Riemann surfaces.

Prerequisites

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Syllabus

Standard Topics — MA 225.

- Analytic function, ∂ and $\bar{\partial}$-derivatives, power series, Laurent series
- Contour integrals, Cauchy theorem
- Removable singularities, zeroes, poles, the maximum principle
- The generalized Cauchy theorem, Runge's theorem
- The resedue theorem, argument principle, Roche's theorem
- Using residues to compute real-valued integrals
- Reflection principle
- Normal families, the Riemann mapping theorem, the Schwarz–Christoffel formula
- Canonical products, the Gamma function, the Riemann zeta function
- the Weierstrass and Mittag-Leffler theorems
- If time permits: $\bar{\partial}$-problem, Cousin problems, harmonic functions, Dirichlet problem.

Additional Topics — MA 226. MA 226 includes one or more of the following topics (instructors choice):

(1) Spaces of analytic functions, relations with Harmonic analysis:
   (a) Hardy spaces: factorization, boundary values, Nevanlina classes
   (b) Interpolation in Hardy spaces: Nevanlina–Pick Theorem, Carleson Interpolation Theorem
   (c) Invariant subspaces of Hardy spaces
   (d) Maximal ideals of H^∞, Corona Theorem, $\bar{\partial}$-equations
   (e) Bergman spaces

(2) Theory of Riemann surfaces and elliptic functions
(3) Introduction to several complex variables

Remark. It is definitely impossible to do all three topics in one semester. Probably the best course of action is to pick one topic and just briefly discuss one of the other two.

Sample Textbooks

- *Functions of One Complex Variable I* (Graduate Texts in Mathematics 11) John B. Conway, Springer-Verlag; 2nd edition (August 1978), 317 pages.
- *Functions of One Complex Variable II* (Graduate Texts in Mathematics 159), John B. Conway, Springer-Verlag; (May 1, 1995), 394 pages.
- *Complex Analysis*, Lars Ahlfors, McGraw-Hill Science/Engineering/Math; 3 edition (January 1, 1979), 336 pages.

Last Updated June 2003

Math 237-238
Partial Differential Equations

Course Catalog Description

The theory of the classical partial differential equations, as well as the method of characteristics and general first order theory. Basic analytic tools include the Fourier transform, the theory of distributions, Sobolev spaces, and techniques of harmonic and functional analysis. More general linear and nonlinear elliptic, hyperbolic, and parabolic equations and properties of their solutions, with examples drawn from physics, differential geometry, and the applied sciences. Generally, semester 2 of this course concentrates in depth on several special topics chosen by the instructor.

Prerequisites

MA 221 and 222.

Syllabus

Standard Topics — MA 237.

- Basic properties of the Laplace, heat and wave equation. These include the explicit solutions of the heat and wave equations in $\mathbb{R}^n$ and a brief review of the maximum principle and related inequalities for elliptic equations.
- First-order equations
- Characteristics
- Basic properties of distributions and Fourier transforms
- Sobolev spaces and inequalities in some detail
- Weak solutions of elliptic equations and their regularity

Standard Topics — MA 238.

- General elliptic operators and boundary conditions
- Statements of general L^p elliptic estimates and Schauder estimates with just a few proofs
- Hyperbolic operators in some generality, the Cauchy problem and characteristics

Additional Topics — MA 238.

(1) Linear topics:
  - parabolic theory
  - semigroups

  - microlocal analysis

(2) Nonlinear topics:
  - nonlinar conservation laws
  - stability theory
  - kinetic theory
  - theory of fluids
  - bifurcation theory
  - variational theory

Sample Textbooks

- For MA237: *Partial Differential Equations* (Graduate Studies in Mathematics, 19), Lawrence C. Evans, American Mathematical Society; (June 1, 1998), 662 pages.

Last Updated June 2003

Math 241–242
Topology

Course Catalog Description

An introduction to algebraic topology with emphasis on cell complexes and manifolds. Topics include fundamental group, covering spaces, homology and cohomology, Poincare duality, and additional topics as time permits.

Prerequisites

Abstract point set topology, or at least metric spaces, including compactness. Basic group theory.

Syllabus

Standard Topics — Math 241-242.

(1) Geometric examples
  - manifolds
  - simplicial complexes
  - CW complexes

(2) Homotopy
  - homotopy equivalence
  - fundamental group
  - categories and functors
  - covering spaces
  - Van Kampen theorem

(3) Homology
  - homology of chain complexes
  - simplicial homology
  - singular homology
  - homology with coefficients
  - Kuenneth formula

(4) Cohomology
  - universal coefficient theorem
  - cup products
  - Poincare duality

Additional Topics for Math 242 (varying from year to year).

(1) Fiber bundles
  - vector bundles and sphere bundles
  - Thom isomorphism and Euler class

(2) Higher homotopy groups
   - Hurewicz theorem
   - Freudenthal theorem
   - Eilenberg-MacLane spaces

Sample Textbooks

- Allen Hatcher, *Algebraic Topology*, Cambridge University Press; 1st edition (November 15, 2001), 550 pages.
- *Elements of Algebraic Topology*, James R. Munkres, Addison Wesley Publishing Company (January 1, 1993), 468 pages, ISBN: 0201627280.

Last Updated August 2003

Math 251-252
Algebra

Course Catalog Description

Basic properties of groups, rings, fields, and modules. Topics include: finite groups, representations of groups, rings with minimum condition, Galois theory, local rings, algebraic number theory, classical ideal theory, basic homological algebra, and elementary algebraic geometry.

Prerequisites

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Syllabus

Standard Topics — MA 251.

(1) Group Theory
   - (a) Review basic group theory
   - (b) First and second laws of isomorphism
   - (c) Solvable and nilpotent groups
   - (d) The Sylow theorems

(2) Ring Theory
   - (a) Introduction to rings
   - (b) Rings, homomorphisms, ideals, maximal and prime ideals, quotient rings
   - (c) Basic examples $\mathbb{Z}$ and $k[X]$

(3) Field Theory
   - (a) Basic properties of fields
   - (b) Algebraic and transcendental extensions of fields
   - (c) Galois theory
     - splitting fields and normal extensions
     - cyclotomic fields
     - fundamental theorem of Galois theory
     - norm and trace
     - solvable groups and solvable extensions
     - unsolvability of the quintic
     - normal basis theorem
     - Hilbert Theorem 90
     - Kummer theory
   - (d) finite fields

(4) Advanced Linear Algebra

   (a) Vector spaces, dual spaces, quotient spaces
   (b) Duality for finite dimensional vector spaces
   (c) Tensor product construction; tensor, symmetric, and exterior algebras
   (d) diagonalization of quadratic forms

Standard Topics — MA 252.

(1) Modules (over not necessarily communtative rings)
   (a) Sub and quotient modules, homomorphisms
   (b) Isomorphism laws
   (c) Finitely generated modules
   (d) Direct sums, direct products, free modules
   (e) Tensor products
   (f) Exact sequences
   (g) Introduction to the Hom functor
(2) Applications
   (a) structure theory of finitely generated modules over polynomial rings and application to Jordan normal form
   (b) representation theory of finite groups up to character theory via group rings. (The latter can be done without the Wedderburn theorems.
(3) Commutative ring theory
   (a) Localization, extension and contraction of ideals
   (b) Localization of modules
   (c) Noetherian rings and modules
   (d) The Hilbert basis theorem
   (e) Integral extensions of rings
   (f) Dedekind domains
(4) Introduction to homological algebra
   (a) Categories and functors
   (b) Projective and injective modules
   (c) Derived functors
   (d) Tor and Ext
   (e) Homological dimension

Additional Topics (as time permits).

(1) Completions
(2) Spec and Proj
(3) Introduction to derived categories
(4) Artin rings
(5) Dimension theory
(6) Wedderburn theory and representations of finite groups

Sample Textbooks

- *Introduction to Commutative Algebra*, M.F. Atiyah, I.G. Macdonald, Addison Wesley Publishing Company (January 1, 1994), 0201407515.
- *Commutative Algebra: with a View Toward Algebraic Geometry*, David Eisenbud, Springer (February 11, 1999), 797 pages, 0387942696.
- *Algebra*, Serge Lang, Springer-Verlag; 3rd edition (January 8, 2002), 912 pages.
- *Linear Representations of Finite Groups*, Jean-Pierre Serre, Springer (October 30, 1996), 170 pages, 0387901906.

Last Updated June 2003

Math 253–254
Number Theory

Course Catalog Description

Introduction to algebraic and analytic number theory. Topics covered during the first semester include number fields, rings of integers, primes and ramification theory, completions, adeles and ideles, and zeta functions. Content of the second semester varies from year to year; possible topics include class field theory, arithmetic geometry, analytic number theory, and arithmetic K-theory.

Prerequisites

Math 251 is required, and Math 252 is helpful.

Syllabus

Standard Topics — Math 253.

(1) The ring of algebraic integers: Dedekind rings, localizations, DVRs, prime ideals and unique ideal factorization
(2) Completions: definitions, Hensel's lemma, unramified extensions and tamely ramified extensions
(3) Ramification, the different, and the discriminant
(4) Cyclotomic fields
(5) Ideal class groups, Dirichlet's unit theorem (for S-units)
(6) Zeta functions and L-functions: basic theory, primes in arithmetic progressions
(7) Group cohomology and Galois cohomology: basics (at least H^0 and H^1)
(8) Ideles and adeles
(9) Beginning of class field theory (if time permits)

Standard Topics — Math 254.

(1) Class field theory: statement of principal results, examples

Additional Topics. Math 254 is primarily a topics course. Examples of topics that have been covered in Math 254 include:

(1) Group cohomology and Galois cohomology: general theory
(2) Diophantine geometry
(3) Arithmetic K-theory
(4) Special values of L-functions
(5) Modular and automorphic forms

Sample Textbooks

- *Algebraic Number Theory* (Graduate Texts in Mathematics, Vol 110) Serge Lang, Springer-Verlag; 2nd edition (August 1, 1994), 357 pages.
- *Number Fields*, Daniel A. Marcus Springer (April 27, 1995), 279 pages, ISBN: 0387902791.
- *Algebraic Number Theory* (Grundlehren Der Mathematischen Wissenschaften 322), Jurgen Neukirch & Norbert Schappacher Springer-Verlag Telos (May 1, 1999), 571 pages.
- *Local Fields* (Graduate Texts in Mathematics 67), Jean-Pierre Serre, Springer-Verlag; 2nd corr. print edition (January 1, 1980), 241 pages.
- *Fourier Analysis on Number Fields* (Graduate Texts in Mathematics, Vol 186), Dinakar Ramakrishnan and Robert J. Valenza, Springer-Verlag.
- *Algebraic Number Theory*, Proceedings of an Instructional Conference Organized by the London Mathematical Society, J. W. Cassels & A. Frohlich , editors, Academic Press; Reprint edition (October 1, 1986).

Last Updated August 2003

Math 263
Probability

Course Catalog Description

Introduces probability spaces, random variables, expectation values, and conditional expectations. Develops the basic tools of probablity theory, such fundamental results as the weak and strong laws of large numbers, and the central limit theorem. Continues with a study of stochastic processes, such as Markov chains, branching processes, martingales, Brownian motion, and stochastic integrals.

Prerequisites

Students without a previous course in measure theory should take MA 221 (or AM 211) concurrently.

Syllabus

Remark. The course is often taken by first year graduate students, who have not yet taken real analysis (they should be taking it concurrently), so the instructor cannot use maesure theory at the beginning of the course.

Standard Topics.

- foundations of discrete probability
- Random walks, Markov chains, branching processes
- random variables and their distribution functions densities
- standard distributions: uniform, Bernoulli, binomial, normal, Poisson, etc.
- conditional expectations (general case)
- convergence of random variables
- Borel–Cantelli Lemma, 0-1 law.
- laws of large numbers
- central limit theorems
- generating functions, characteristic function, and their applications

Additional Topics (as time permits).

- Introduction to random processes
  - stationary processes
  - Wiener processes
  - martingales

Sample Textbooks

The limited background of many students (see the earlier remark) severely limits the choice of a text; the course requires a sufficiently high level book that does not immediately start with Kolmogorov's definition of probability spaces.

- *Probability and Random Processes*, Geoffrey R. Grimmett & David R. Stirzaker, Oxford University Press; 3rd edition (August 1, 2001), 608 pages.

Last Updated June 2003