MATH 285y TROPICAL GEOMETRY SPRING 2013
PROBLEM SET 3, DUE THURSDAY APRIL 11

1. (a) (Textbook 1.9) Compute and draw the tropical quadric in $\mathbb{R}^2$ passing through the points

$$(0,5),(1,0),(4,2),(7,3),(9,4).$$

   (b) (Textbook 1.4) Show that there is a unique tropical quadric

$$A \odot X^{\odot 2} \oplus B \odot X \odot Y \oplus C \odot Y^{\odot 2} \oplus D \odot X \oplus E \odot Y \oplus F$$

   passing *stably* through any five points $(x_1, y_1), \ldots, (x_5, y_5) \in \mathbb{R}^2$, even if they are in special position. What should stable containment mean here? Can you give a formula for the coefficients A, B, C, D, E, F in terms of the points?

2. Let $X \subset \mathbb{C}[x_{11}, \ldots, x_{33}]$ be the variety of 3×3 matrices of rank at most 2. Use `gfan` to compute $\text{Trop}(X)$. Give the dimension, lineality space, and *f-vector* of this tropical variety. How much of this data can you interpret combinatorially?

3. (Textbook 1.14) Consider the plane curve given by the parametrization

$$x = (t-1)^{13} t^{19} (t+1)^{29} \text{ and } y = (t-1)^{31} t^{23} (t+1)^{17}.$$

   Find the Newton polygon of the implicit equation $f(x,y) = 0$ of this curve. How many terms do you expect the polynomial $f(x,y)$ to have?

4. (Textbook 1.12) The amoeba of a curve of degree four in the plane $\mathbb{C}^2$ can have either 0, 1, 2 or 3 bounded convex regions in its complement. Construct explicit examples for all four cases.

5. Let C be the metric graph with two vertices and four parallel edges between them of lengths $a, b, c, d > 0$. Compute the Jacobian of C. Draw the image of C in $\text{Jac}(C)$ under the Abel-Jacobi map, and draw the theta divisor $[\Theta]$.