Math 123 HW 10

1. In class and in the tribones notes I described the transplantation process as one in which you draw a path in the first hexagonal tiling, read off the labels, and then redraw it in the second tiling according to the string you get. Suppose you were do redefine the process switching the roles of the two labelings. Does this always give you the same answer (up to translation)? Either prove it or give a counter-example.

2. Prove that the Heisenberg graph is not quasi-isometric to $\boldsymbol{R}^3$. You'll probably want to use the fact that the balls of radius N in the Heisenberg group have on the order of N^4 points in them.

3. Say that a countable graph (with the natural numbers as a vertex set) is *reasonable* if it is connected and the following is true:

- Every vertex is incident to infinitely many other vertices.
- Every vertex is not incident to infinitely many other vertices.

Prove that there infinitely many pairwise non-isomorphic reasonable graphs.