Math 123 HW 5

1. This is problem 6.2.5 from the book. Determine the minimum number of edges you need to delete from the Peterson graph in order to make it planar.

2. This is problem 6.2.8 from the book. A graph is called *outerplanar* if it has a planar drawing in which all the vertices are on the outer face. Figure 1 shows an example. Use Kuratowski's Theorem to prove that a graph is outerplanar if and only if it does not contain a subdivision of $K_{2,3}$ or K_4.

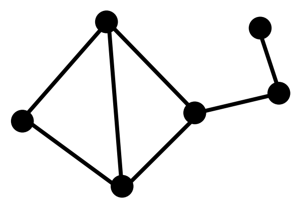

Figure 1: An outerplanar graph

3. Let G be a 2-connected planar graph. (This means that all faces of G are bounded by cycles.) Let $T \subset G$ be a spanning tree. Let G^* denote the dual graph of G. Let T^* be the subgraph of G^* consisting of edges which cross the edges of $G - T$. Prove that T^* is a tree. Figure 2 shows an example.

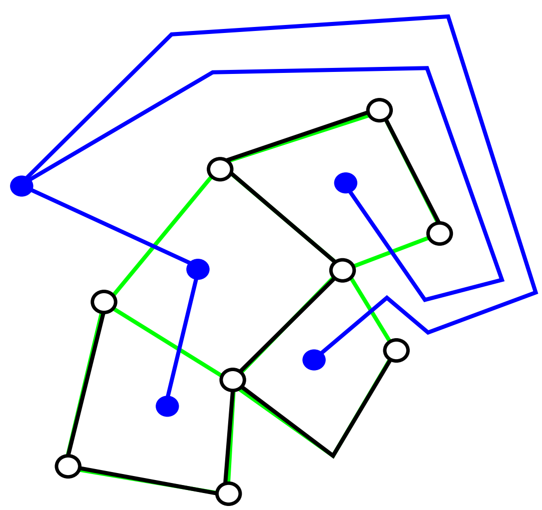

Figure 2: An outerplanar graph

4. Say that a *triangulation* of a surface S is a graph drawn on S such that all the faces are (planar) triangles. Prove that a triangulation of a surface of genus g cannot be drawn on a surface of genus $g' < g$. For instance, K_7 can be drawn as a triangulation of the torus but it cannot be drawn on the sphere.

5. Formulate a definition of a 3 dimensional space which is a union of finitely many tetrahedra and has properties analogous to a compact surface. What you want is that every point in the space is the center of a ball which intersects the space in a subset that is homeomorphic to a 3-dimensional ball. So, in other words, if M is your space and $p \in M$ is a point, then there is a ball B centered at p and a continuous bijection $f : B \cap M \to B_0$ whose inverse is also continuous. Here B_0 is the standard open unit ball in $\boldsymbol{R}^3$. If you think about the definition of a compact surface this is what you get, except for disks rather than balls. These kinds of spaces are usually called *simplicial 3-manifolds*. You'll want a condition for edges, faces, and vertices.

Challenge Problem: Prove that there is a graph that can be drawn on the Klein bottle but not on the torus.