Math 123 HW 8

1. This is problem 3.1.8 from the book. Prove or disprove: Every tree has at most 1 perfect matching.

2. This is problem 3.1.18 from the book. Two people play a game on a graph G, alternately choosing distinct vertices. Player 1 chooses any vertex. Player 2 then chooses a vertex adjacent to the last choice made by Player 1. Player 1 then chooses a vertex adjacent to the last choice of Player 2. And so on. The last player able to move wins. Prove that Player 2 has a winning strategy if and only if G has a perfect matching.

3. This is problem 3.1.31 from the book. Prove Hall's Theorem using the Konig-Egervary Theorem. (This is the result that the size of the maximum matching equals the size of the minimum vertex cover.)

4. Let G be any finite group. Prove that there exists a graph Γ so that the group of automorphisms of Γ is isomorphic to G. Hint: Fool around with the Cayley graph $\Gamma(G, S)$ where S is a well-chosen set of generators of G.

5. Consider the Cayley graph $\Gamma(\boldsymbol{Z}, F)$ where F is the set of Fibonacci numbers. Prove that this graph has infinite diameter. This is really a question about elementary number theory.

6. Let T_k be the infinite regular tree having degree k. Prove that T_3 and T_4 are quasi-isometric. This means that there is a constant K and a map f from the vertices of T_3 to the vertices of T_4 (not necessarily one to one or onto) with two properties:

- $(1/K)d_3(a,b) - K < d_4(f(a), f(b)) < Kd_3(a,b) + K$.
- For every $b \in T_4$ there is some $a \in T_3$ such that $d_4(f(a), b) < K$.

Here d_k denotes the usual path distance on T_k.