Math 1410: HW 11:

1. Prove that the connected sum of a torus with the projective plane is homeomorphic to the connected sum of 3 projective planes. (Hint: play around with hexagon gluing diagrams.)

2. Let Γ be a finite graph embedded in $\boldsymbol{R}^3$ in such a way that all the edges are line segments and all vertices have degree 1 or 3. Let $N_\epsilon(\Gamma)$ be the set of points $p \subset \boldsymbol{R}^3$ within ϵ of Γ. Prove that the boundary of $N_\epsilon(\Gamma)$ is a surface provided that ϵ is sufficiently small.

3. Prove that any finite connected sum of tori is homeomorphic to the kind of surface defined in problem 2.

4. Let Σ be and connected sum of 2 tori, made from an octagon gluing diagram. Let p be the center of the octagon. Prove that there is a continuous unit vector field on $\Sigma - \{p\}$.

5. Think of the projective plane as an octahedron with its antipodal points identified. This gives a triangulation of the projective plane with 4 triangles, 6 edges, and 3 vertices. Compute the simplicial homology of the projective plane with respect to this triangulation, using both $\boldsymbol{Z}$ and $\boldsymbol{Z}/2$ coefficients.

6. Triangulate the connected sum of $n \geq 2$ tori as you like and compute its simplicial homology with $\boldsymbol{Z}$ coefficients. Hint: start with $n = 2, 3$ and see if you can find the pattern.