Math 20 Midterm 1. 12 Oct 2010

Instructions. The problems are worth 25 points each. Show all your work.

1. Consider the curve $r(t) = (t, t^2, t^3)$.

a. (10 pts) Find $\sin^2(\theta)$, where θ is the angle that the tangent vector to the curve makes with the x-axis at time $t = 1$.

b. (15 pts) Determine whether the tangent lines at $t = 1$ and $t = 2$ are parallel, intersecting, or skew. Explain your answer.

2. Give an example of a differentiable function $f(x, y)$ such that...

a. (15 pts) the gradient of f vanishes at $(0, 0)$ but f does not have a local max or min at $(0, 0)$.

b. (10 pts) f has a minimum at $(0, 0)$ but f fails the second derivative test at $(0, 0)$

Explain your answer in both cases.

3. Consider the function $f(x, y, z) = xy + y^2 + z^3$. Let $r(t) = (x(t), y(t), z(t))$ be a curve such that goes through the point $(1, 1, 1)$ at $t = 0$. What is the slowest possible speed r could have at time $t = 0$ so that

$$\frac{d}{dt} f(r(t)) = 1?$$

4. Find the point on the parabola $y = x^2$ that is closest to the line $y = x - 1$. Hints: First of all, draw a good picture. It is possible to do this problem with hardly any calculation, but if you find the algebra getting messy, you might want to set the problem up as a Lagrange multipliers problem.