Math 271 Qual Credit I:

1: Give a careful proof that the Klein and Poincare models of hyperbolic space are isometric, when the metrics are suitable scaled. You can't just appeal to Hadamard's Theorem.

2: Give a careful proof, with definitions included, that the Teichnuller space of marked hyperbolic structures on a genus 2 surface is homeomorphic to $\boldsymbol{R}^6$.

3: Let $h : \boldsymbol{C} \cup \infty \to \boldsymbol{C} \cup \infty$ be a quasi-conformal map, as defined in class. Suppose $h(\infty) = \infty$. Let $Q \subset \boldsymbol{C}$ be a square in the plane. Prove that there is a constant K and disks $D_1 \subset h(Q) \subset D_2$ such that $\text{diam}(D_2)/\text{diam}(D_1) < K$. The constant K is supposed to be independent of Q.

4: Look up the usual definition of quasi-conformal maps, in terms of extremal length or conformal modulus, and prove that it is equivalent to the definition given in class.

5: Suppose you have a labeling of the edges of the dodecahedron by elements of $\{+, 0, -\}$ suppose that for every pentagonal face there are at least 2 sign changes as you go around. Prove that all the labels are 0. This is a special case of the general argument used in Cauchy Rigidity.

6: Recall that in class I gave two definitions of the translation surface associated to a polygon with rational angles. One definition involved a cut-and-paste argument and the other one involved the universal cover and the holonomy map. Prove that the two definitions are equivalent.

7: Work out the Veech group for the translation surface made by gluing opposite sides of a regular 12-gon.

8: Let $f : \boldsymbol{R} \to \boldsymbol{CH}^2$ be a bi-lipschitz embedding. Here $\boldsymbol{CH}^2$ is the complex hyperbolic plane. Prove that there is some geodesic γ in $\boldsymbol{CH}^2$ such that $f(\boldsymbol{R})$ stays within a bounded distance of γ.