Math 271 Qual Credit II:

1. Let $\mathcal{M}(4,4,4)$ denote the moduli space of isometry classes of $(4,4,4)$ triangles in the complex hyperbolic plane. Prove that $\mathcal{M}(4,4,4)$ contains infinitely many points.

2. Prove that complex hyperbolic space does not have constant sectional curvature. Hint: If you find yourself making calculations with the Riemann curvature tensor, you are doing the problem incorrectly. The main point is to consider the different totally geodesic slices.

3. Suppose that P is a convex pentagon. One can do a butterfly move based on each of the 5 sides of P. This gives rise to a group action of $(\boldsymbol{Z}/2)^5$ on the hyperbolic plane. Describe the set of pentagons P for which this this group is discrete, with compact quotient.

4. Suppose that P_1 and P_2 are 2 convex n-gons which are affinely equivalent. That is, there is an affine transformation T such that $T(P_1) = P_2$. Let G_j be the hyperbolic reflection group based on the butterfly moves on P_j. Prove that G_1 and G_2 are conjugate groups. That is, there is a hyperbolic isometry I such that $IG_1I^{-1} = G_2$.

5. Prove that any embedded polygon can be triangulated so that each edge in the triangulation connects a pair of vertices of the polygon.

6. Let $\mathcal{M}$ denote the moduli space of labeled flat cone structures on the sphere with angle deficit list $\pi/2, \pi/2, \pi/2, \pi/2$. So, two elements of $\mathcal{M}$ are close if and only if there is a near-isometry from one to the other which preserves the labels. We know that $\mathcal{M}$ is a lattice quotient $\boldsymbol{CH}^1/\Gamma$ of finite area. That is, $\mathcal{M}$ is a hyperbolic surface. Which surface? Explain your answer.

7. Prove that every two cone points on a flat cone sphere can be connected by a line segment which does not go through any other cone point.

8. Let Δ be the interior of the regular simplex, equipped with the Hilbert metric. Prove that Δ is quasi-isometric to $\boldsymbol{R}^3$.