Math 52 Sample Midterm

1. Find all solutions to the following equation, using any method you like. (But show your work.)

$$3x_1 + 7x_2 + x_4 = 3$$

$$11x_2 - 8x_4 = 9$$

$$17x_2 - 3x_3 - x_1 = 5.$$

2. Consider the 5×5 matrix

$$M = \begin{matrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & a & 0 & 0 \\ 0 & b & 1 & c & 0 \\ 0 & 0 & d & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{matrix}$$

Find values of a, b, c, d, all nonzero and all different from each other, such that A is not invertible.

3. Find the eigenvalues and eigenvectors of the matrix

$$M = \begin{matrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{matrix}$$

4: Let M be a square matrix. Prove that $(M^t)^{-1} = (M^{-1})^t$. In other words, the inverse of the transpose of M equals the transpose of the inverse of M.