# LINEAR RESOLVENT GROWTH OF RANK ONE PERTURBATION OF A UNITARY OPERATOR DOES NOT IMPLY ITS SIMILARITY TO A NORMAL OPERATOR.

NIKOLAI NIKOLSKI AND SERGEI TREIL

*To the memory of Tom Wolff*

**Abstract.** The main result of this paper is that for any unitary (selfadjoint) operator $U$ with non-trivial absolutely continuous part of the spectrum there exists a rank one perturbation $K = ba^* = (\,\cdot\,, a)b$, such, that the operator $T = U + K$ satisfies the Linear Resolvent Growth condition (LRG),

$$\|(\lambda I - T)^{-1}\| \le C/\operatorname{dist}(\lambda, \sigma(T)), \qquad \lambda \in \mathbb{C} \setminus \sigma(T),$$

its spectrum lies on the unit circle $\mathbb{T}$ (on the real line $\mathbb{R}$), but $T$ is not similar to a normal operator.

This contrasts sharply with the result of M. Benamara and the first author that if a finite rank perturbation $T = U + K$ of a unitary operator is a *contraction* ($\|T\| \le 1$), then it is similar to a normal operator if and only if it satisfies (LRG) and its spectrum does not cover the unit disc $\mathbb{D}$.

*Date*: July, 2001.