

COMMON COMPLEMENTS OF TWO SUBSPACES OF A HILBERT SPACE

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Abstract: In this paper we find a necessary and sufficient condition for two closed subspaces, \mathcal{X} and \mathcal{Y} , of a Hilbert space \mathcal{H} to have a common complement, i.e. a subspace \mathcal{Z} having trivial intersection with \mathcal{X} and \mathcal{Y} and such, that $\mathcal{H} = \mathcal{X} + \mathcal{Z} = \mathcal{Y} + \mathcal{Z}$.

Unlike the finite dimensional case the condition is significantly more subtle than simple equalities of dimensions and codimensions, and non-trivial examples of subspaces without a common complement are possible.

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